

Turnover Costs: Evidence from Unexpected Worker Separations*

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Abstract

To what extent do unexpected worker exits affect firm profits? To answer this question, we exploit exogenous variation coming from unexpected worker deaths, linked with rich administrative data on firm profitability in Denmark. We find that turnover costs are large – equal to roughly one year of labor costs for an average employee – much larger than previous evidence based on employer surveys. The imperfect substitutability of incumbent workers with outsiders drives the larger estimates. We find negative effects on revenue even after employment has recovered and higher labor costs to retain the remaining workers. Our findings of substantially higher replacement costs help to resolve puzzles in search and matching models, which struggled to reconcile seemingly low turnover cost estimates with several other empirical regularities.

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1 Introduction

The majority of workers voluntarily leave their jobs. For each worker who is laid-off, another three decided to quit voluntarily in September 2022 in the US. Despite the prevalence of worker-initiated separations, there is little evidence on the effect of such separations on firm outcomes.¹ What are the consequences on firms' revenue and the costs of such events? Do they also impact coworkers' hours worked and wages? Answers to these questions would help to understand the economic consequences of one of the most salient labor market phenomena. The size of turnover costs also matters to explain the observed cyclical volatility in labor market variables in unemployment theories or the optimal design of advance notice.

Few studies provide estimates of turnover costs. Indeed, papers summarizing knowledge on these questions note the lack of evidence and attribute it to the difficulty of measuring the consequences of quits (see, e.g., Manning 2011). Most evidence stems from employer surveys regarding the perceived explicit costs of recruiting and training workers. These studies suggest that turnover costs are small, corresponding to less than a month of an individual worker's salary.

There are several reasons to believe that survey-based evidence could underestimate the actual turnover costs. First, the magnitude of estimates is in contrast with the mirror event, that is, forcing an employee to leave her job. In fact, earnings losses associated with job displacement are large (see, e.g., Jacobson et al. 1993; Bertheau et al. 2022). Second, existing evidence capture only part of turnover costs, which may comprise components other than direct recruiting and training costs. Furthermore, they do not measure turnover's actual consequences but interviewees' assessments.

An ideal experiment would force workers to quit and study the consequences for firms. This requires exogenous variation in quits and rich data on firm outcomes. To mimic as much as possible such an experiment, we exploit Danish data and use unexpected deaths as a plausibly exogenous shock. This research design allows us to overcome existing challenges in the literature by measuring the costs to the firm of involuntary employee turnover due to unexpected employee death and hence provide an estimate of turnover costs. The use of worker deaths as an exogenous variation has been used in recent papers to shed new light on a variety of research questions (see, e.g., Azoulay et al. 2010; Jäger and Hening 2022). We draw on matched employer-employee data linked with firm's financial account data and study the consequences of unexpected deaths by comparing treated firms to a comparison set of firms with similar characteristics before the event. Unexpected deaths are defined as all workers

¹The importance of quits is not specific to the Great Resignation. They represent the majority of separations in most countries (Cahuc et al., 2014).

who die, do not receive any social transfers sickness, and do not spend any day at hospitals in the last two years before the year of the death. We excluded all deaths at work and firms with more than one deceased in a year.

The research design is an event study. We match firms based on lagged observable characteristics to find a similar group of firms that did not ultimately experience a worker death in the (placebo) event year. Our empirical design further enables us to implement tests of our main identifying assumption by assessing whether trends of relevant outcome variables are similar, comparing treated and matched comparison establishments up to three years before the event. To better understand our results, in the second part of the paper, we use a search and matching model that rationalizes our empirical results.

The sample, constituted of more than 1,700 unexpected worker deaths, is representative of an average employee. The most common causes of death are premature events, such as accidents, and we exclude deaths at work. Most deceased workers have a high-school diploma and are male, with four years of tenure in the firm where he works. A treated firm has, on average, 13 employees. The coworkers of the deceased workers have similar characteristics to coworkers of the placebo deceased workers in control firms. To ensure that our results are not driven by business closure, we restrict our sample to firms present three years before and four years after the event. We focus on small firms, from 3 to 30 workers in all private sectors of the economy.

Denmark offers an ideal environment to study this question. The Danish legal and institutional constraints on employers are more similar to those in the US than in other continental European economies. Denmark has no specific hiring and firing regulations, and employees must give their notice periods a few weeks before quitting. Practically, Denmark is one of the rare countries in the World where researchers can combine administrative data that link employer to employee, firm's financial account, and data on deaths and hospitalization events.

Our main finding is that unexpected separation due to worker deaths subsequently reduces firms' profits by roughly one year of salary for the average employee. This estimate corresponds to a profit reduction of 41,000 EUR accumulated over the four years after a worker's death. Our estimate of turnover costs is thus an order of magnitude larger than previous studies based upon survey evidence on recruiting and training workers. The previous literature has found that the costs of recruitment and training are equal to roughly one month of salary for the average worker. For example, on a survey of Californian firms, Dube et al. (2010) find that replacement costs are, on average, 4,000 US dollars overall. They estimate a turnover cost about 2,000 US

dollars for blue-collar and manual labor workers and 7,000 US dollars for professional and managerial employees. In contrast, our estimates imply turnover costs that are an order of magnitude higher at about one year.

What drives our higher estimated turnover costs? We show that two effects play an important role: the revenue net of intermediate inputs costs (value added) and the wage bill. These costs are typically not captured by survey studies, and they substantially increase turnover costs. First, we show long-lasting and large negative effects on value added. Losing an employee to premature death lowers value-added in the subsequent four years by a total of EUR 35,000. Second, we show that incumbent workers' earnings increase by about EUR 25,000. However, the effect on the total wage bill is substantially smaller as newly hired workers, in response to the shock, start at lower salaries than the deceased worker.

We conduct a heterogeneity analysis across subsamples to understand the mechanisms behind our results. The effect of tenure is inverse-U shaped as low-tenure workers may be easier to replace from the firm's perspective. In contrast, the loss of a long-tenured worker may include workers closer to the retirement age for which the firm already has a contingency plan. These results align with some of the predictions of the theoretical frameworks of Lazear (1979) and Becker (1964). On the one hand, in the incentive explanation put forward by Lazear (1979), high-tenure workers are overpaid; therefore, losing such a worker leads to lower profit losses. On the other hand, in the firm-specific explanation put forward by Becker (1964), high-tenure workers are underpaid; therefore, losing such a worker leads to higher profits. Also, we find that high-paying and more productive firms rely on more skilled workers; hence, losing a worker may have larger consequences. Taken together, our evidence suggests that worker skill is a key component driving turnover costs.

Finally, we use a theoretical model to understand how a firm's technology and co-workers' preferences impact turnover costs. Our benchmark model features heterogeneous firms in a search-and-matching framework with employment contracts determining wages and hours worked. The model builds on Cooper et al. (2007) but differs in subtle assumptions. In the model, a worker separation induces a drop in revenue for the firm and increases incumbent workers' hours. The model's predictions for the effects of worker exits thus line up qualitatively with our reduced-form evidence. We then show how the reduced-form effects directly depend on the model's structural parameters and can thus be used to identify the parameters of interest cleanly. Indeed, in the search and matching framework, effects on profits capture turnover costs. Our model delivers three estimable parameters.

The cost of adjusting the hours of incumbent workers depends on the elasticity of

their labor supply and the sensitivity of wages to work hours. We show that the adjustment at the internal margin dampens the cost of unexpected workers' exits, meaning that lacking attention to this adjustment margin may lead to overestimating turnover costs. We use instrumental variables to identify these three parameters. Our structural parameter estimates reveal that workers' earnings adjustments and hours responses of incumbents matter to explain turnover costs.

The main contribution of this paper is to provide novel estimates of the costs to the firm of unexpected worker exits. These exits are pervasive: quits are the most common reason for job termination, and many are not expected (Garland, 2016; Mercan and Schoefer, 2020). Up until now, empirical evidence is drawn from the departures of highly qualified workers or case studies (see, e.g., Kuhn and Yu 2021, Bennedsen et al. 2020, Huber et al. 2021). In contrast, we provide evidence from a representative sample of workers. These estimates have implications for the search and matching literature that single out the hiring decision as a key parameter to understand unemployment fluctuations. Indeed the fixed cost of recruitment in an otherwise standard search and matching model reduces the sensitivity of hiring cost to labor market tightness, allowing vacancies to rise more in response to a positive shock (see, e.g., Pissarides 2009). Wasmer (2006) builds a model that can explain differences in turnover costs across countries. Bloesch et al. (2022) build a model with the implication that the output loss from sudden turnover is higher when workers acquire position-specific skills on the job. We contribute to a nascent literature that uses unexpected worker deaths as plausibly exogenous variation in worker separation. Due to data limitations, existing studies have not yet studied the impact of such shocks on firm outcomes. Jäger and Hening (2022) show that unexpected deaths raise the remaining workers' wages, and the average effect masks substantial heterogeneity.

The paper is organized as follows. We describe the institutional context, data, and empirical strategy in Section 2. Section 3 provides quasi-experimental estimates of unexpected worker exits on firm and incumbent worker outcomes. Section 4 provides estimates of structural parameters. Section 6 concludes.

2 Context, Data, and Empirical Strategy

Our research design aims to approximate the ideal experiment of randomly terminating employment relationships and examine how the pay-offs of employer and worker change. After describing the relevant features of the Danish labor market and the data, we define the treated group of firms and the comparison group.

2.1 Context: The Danish Labor Market

In a broad comparison, the features of the Danish labor market are more closely related to the US than to continental Europe. Unlike many European countries, the Danish labor market is characterized by the absence of restrictive regulations on hiring and firing. This feature leads to one of the highest levels of job mobility in the OECD (see Figure A.1). According to several other Employment Protection Law (EPL) indexes, Denmark has among the most flexible employment protection laws among advanced economies.

We study small firms that have less than 30 employees. Those firms represent the vast majority of the population of firms and a large share (around 35%) of total employment (as in other OECD countries, see OECD 2017).

Like most European countries, the Danish collective bargaining agreement is based on a two-tier structure, with sectoral bargaining of wage floors followed by local bargaining at the firm level. However, the pay of 80% of workers is established through local negotiations at the firm level (see Table A.1). Industry-level agreements are limited to other working conditions. For the remaining 20% of the workers, the industry agreements set out the base pay. In this case, firms can easily deviate from wage floors by setting pay supplements.

In all, the Danish labor market provides a relevant institutional setting to study the effects of worker exits on firms.

2.2 Data to Identify Firm-Specific Labor Supply Shock

To detect quasi-experimental variation in firms' employment, we use matched employer-employee data from 1990 until 2013, which we link to several administrative data sets containing information on firms' financial accounts, death events, hospitalization events, and social transfer payments. Below, we present the relevant characteristics of the data sets.

The backbone data set is a linked employer-employee data that measure key information on all employment relationships at the annual frequency (IDAN, *IDA ansættelser*). In particular, we measure occupations, job tenure, labor earnings, and hours worked. Occupations are coded following the International Standard Classification of Occupations (ISCO). Labor earnings include all taxable income, that is, it includes overtime and bonus pay. Hours data up to 2007 are bounded at 27 hours per week. From 2008 onwards, employers must declare *paid* hours worked for each job at the monthly frequency.

To define firm-level outcomes, we use a data set containing financial data such

as value-added, capital stock, and total labor costs (FIRM, *Generel firmastatistik*). The firm's financial data set covers some sectors from 1992-1998 and all sectors since 1999. We use additional information such as the number of employees, years in business, and the industry classification (at the 4-digit level) to define the comparison group.

To define the treatment, we use data sets that record the date, the cause, and the location of death events (DOD, *Døde i Danmark* and DODSAASG, *Dødsårsagsregister*). To ensure that the death is unexpected, we use data on hospitalization events (LPR, *Landspatientregistret*) and social transfer payments (DREAM, *Beskæftigelsesministeriets forløbsdatabase*).

To better understand coworkers' responses to the negative labor-supply shock, we use data that differentiates between agreed-upon hours and overtime hours and agreed-upon hourly pay and realized pay. The data are from a mandatory employer survey covering companies with at least ten employees since 1999 (LONN, *Lønstatistikken*). The survey quality is high, as it is used in Eurostat data sets.

An overview of data sources, variables, and sample construction necessary for replication is reported in Appendix B.

2.3 Sample Restriction and Definition of Treated Firms

Firms can be treated from the calendar years 1993 to 2010. We select treated and comparison firms and follow them seven years, three years before and after around an event/relative year k . Hence the event window is $k \in (-3, 3)$.

We restrict our analysis to active firms within the event window. Firms are defined as active when both employment and sales are positive in a given year. Also, we restrict our analysis to private sector firms employing 3 to 30 workers for the last 3 years before the year of the death event.

Unexpected deaths are all deaths of workers who do not receive any social transfers and do not spend any day at hospitals in the last two years before the year of the death. These restrictions are stringent as social transfers include unemployment insurance, parental benefits, and sickness benefits. The aim is to exclude deaths that employers could have expected. We exclude deaths that occur at work as they could potentially be related to bad workplace practices. Also, we exclude the deaths of workers that are classified as employers. Treatment status is defined as one unexpected death.

Finally, incumbents are employed workers for three years and up to at least the date of the reported death. Contrary to the firm sample, we do not condition on any outcomes after the event. Our worker panel is, therefore, unbalanced.

2.4 Selecting Comparison Firms

Unexpected worker deaths are rare events. While it has the downside of having relatively few treated firms, it has the upside that we have a large pool of comparison firms. We select a set of comparison firms with similar characteristics compared to firms in the treated group based on lagged characteristics. More precisely, each firm in the treatment group is matched to a firm with similar characteristics. Several papers use this method to estimate causal effects (see, e.g., Jäger and Hening, 2022)).

Treated firms are matched to similar firms based on their characteristics one year before the event. This procedure allows us to check the evolution of key outcome variables before the event. The exact matching is implemented using the characteristics of the deceased worker – the deciles of age and hourly wages – and firm characteristics – the number of employees and sector (two-digit level).

For cells with multiple matches for each treated firm, we select the most comparable firm based on a propensity score. The propensity score is estimated using a linear probability model. To ensure that treated and comparison firms have similar characteristics, we match on a rich set of characteristics to estimate the propensity score. Precisely, we include in our model characteristics of the workforce (the percentage of men and average age), the number of employees, the average wage paid, hiring and separation rate, value-added, capital stock, investments, and 4-digit industry fixed effects.

2.5 Summary Statistics

On average treated and control firms have 13 employees, with around 7 hires and separations each year. Value added (revenue net of intermediate inputs) and profit (value added net of labor costs) are very similar across groups (see Table 1). The mean value added is around 590K€, and the profit amounts to around 180K€.

In the treated and comparison groups, incumbent workers earn 35K€, have 12 years of education (corresponding to vocational training), are male at 74%, and are 44 years old on average. The attrition is the same in both groups: incumbent workers are present in the data around 6.81 out of 7 years (see Table 1).

The median deceased worker is 57 years old at the moment of the death and worked full-time the year before the event. He has been employed for at least 4 years in this firm (see Table A.2).

Additional specifications. We find similar results when we pool the three years preceding the events and not solely the preceding year (see Tables A.15 and A.16). Table

A.17 reports the balancing analysis for all incumbent workers. The entire sample of workers is younger and more educated than incumbent stayers.

2.6 Difference-in Differences specification

We estimate the effect of worker exits on several outcomes at the firm level using the following dynamic difference-in-differences specification:

$$y_{jk} = \alpha + \gamma_j + \sum_{k=-3}^3 \beta_k \mathbb{1}(Period_k) + \sum_{k=-3}^3 \beta_k^T \mathbb{1}(Period_k) Treated_j + \epsilon_{jk}, \quad (1)$$

where y_{jk} denotes the outcome of firm j in event year k . γ_j is an event-firm fixed effect. $Period_k$ are dummies equal to one the relative to the event period k , and zero otherwise. Event-firm fixed effects imply that firms sampled in two different events will be coded as two different firms. We use the year preceding the event ($k = -1$) as the reference period. $Treated_j$ equals one for all periods for firms in which an employee dies at $k = 0$ and is zero otherwise. β_k captures baseline trends around each event. The parameters of interest, β_k^T , measure the effect of a worker death for firms in the treatment group relative to the reference year, $k = -1$. The event-firm fixed effects absorb permanent differences between the treated and comparison firms. Hence, the variation used for identification stems from within-firm variation over time relative to the event. The leads and lags around the event allow us to evaluate the common trends assumption underlying our research design - that treated and comparison groups would have followed parallel trends in the absence of the worker death, the plausibility of which we test in the pre-period. Finally, standard errors are clustered at the event-firm level. Our design is not subject to additional assumptions required in DiD design subject to negative weights (see de Chaisemartin and D'Haultfoeuille (2020)). Indeed, the share of firms with one worker who dies in control of treated groups before or after the year of the event is zero. Besides, we do not need to include year-fixed effects as the one-to-one matching lets calendar time orthogonal to the treatment.

We estimate a similar specifications at the incumbent worker level:

$$y_{ik} = \alpha + \gamma_i + \sum_{k=-3}^3 \beta_k \mathbb{1}(Period_k) + \sum_{k=-3}^3 \beta_k^T \mathbb{1}(Period_k) Treated_i + \epsilon_{ik}, \quad (2)$$

where γ_i is a worker fixed effect.

3 Effects of Worker Deaths on Employment, Profits and Coworker Wages

3.1 Firm-Level Results

Effect on firm employment. We start by looking into the effect of one additional separation due to worker deaths on the number of separations and hires (see Figure 1, Panel (a)). We find flat pre-trends for both variables, suggesting that treated and comparison group firms were on similar trajectories. We find a sharp increase in separations of 1.04 workers during the event year. This magnitude corresponds to the mechanical effect we would expect if one worker death triggers one additional separation in the year it occurs. The estimated coefficient could still differ as the worker death could trigger additional separations or lead to fewer ones. In fact, we find a (statistically not significant) reduction of 0.21 separations in the post-event year. In the following years, we do not detect effects on separations. One worker's death translates into about 0.6 additional hires within a relatively short period. We find small and positive but statistically not significant estimates in the following years.

We find completely flat pre-trends and a negative statistically significant drop of 400 hours worked at the firm level during the event year (see Figure 1, Panel (b)). In the following years, we find a reduction between 60 to 240 hours. Importantly this differs from the incumbent coworker hours effects since we here take into account not only the incumbent workers but all employees, including those who newly joined the firm. We will further dissect the result below when we turn to incumbent coworkers. Consistently with the results reported above, we find a reduction of 0.3 workers at the end of the event year and the following year when we use the number of employees as the outcome variable (the mean is 13 employees before the event). Two years after the event, we still find negative effects of -0.2, but the confidence interval includes zero. After three years, total employment at firms affected by worker death has converged to the comparison group trend (see Figure A.2). Table 2 reports the point estimates and standard errors. Overall, our results suggest that the deceased worker is not immediately replaced. Still, after some years, we do not detect a difference in the level of labor input following the negative labor supply shock.

Effect on firm profits. Next, we investigate the effect on firm profits. The point estimates indicate that profits fall between EUR 10,000 and EUR 20,000 during the event year and the following years (see Figure 2, Panel (a)). Note that the standard errors are relatively large, but exclude zeros in some years. Using these point estimates, can

directly estimate the turnover costs: $\hat{\Pi} = \hat{\beta}_0^T + \hat{\beta}_1^T + \hat{\beta}_2^T$, where $\hat{\beta}_{0,1,2}^T$ refers to the estimates from the difference-in-differences model (1). Hence, the cumulative negative effect on the profit of losing one employee amounts to 41,000 EUR. Table 1 Panel (b) reports that the annual labor earnings of incumbent workers are about 35,000 EUR. Therefore, we find that unexpected worker exits lower profits by more than one year of an annual salary. Our research design allows us to provide novel evidence on the cumulative profit losses associated with unexpected exits.

We decompose profit losses into two terms: value-added and labor costs (see Figure 2, Panel (b)). We find flat pre-trends in the pre-event periods. During the event year, we estimate a drop of -10,000 EUR, relative to a control mean of 660,000 EUR, followed by persistently negative effects on the order of magnitude of 10,000 EUR in the following years. Standard errors are of a similar order of magnitude so we cannot rule out that the overall effects are zero. Table 2 reports the point estimates and standard errors.

Additional results We report additional results in Table A.3, A.4, A.5, and A.6. Table A.3 decomposes the impact on profit between revenue, intermediate input costs and labor costs. The year of the event, we observe a drop of total revenue (about 15,000 €) as well as a drop of intermediate input costs (about 10,000 €). In the following year, total revenue is still below the level of comparison firms (about 6,000 €) but intermediate input costs is *above* (about 5,000 €). The gap between revenue and cost generate the persistent decline in profit. Table A.4 reports effects for financial variables and decomposition of total earnings and total hours worked. We do not find an effect on capital, total assets, equity and value of export. We do find a statistically significant increase of about 10,000 € on *overtime earnings*. Table A.5, and A.6 provide some heterogeneity analysis by characteristics of firms and the deceased workers. We provide evidence on *short-run* effects, that is the effect for the year of the event and the following year ($k = 0$ and $k = 1$). The negative effect of worker death on number of employees is stronger for small, and low-productive firms (table A.5, column 1 and 3). For heterogeneity related to the characteristics of the deceased worker, we find that low-earnings and older workers are less replaced. The negative effect on the net profit is stronger for small firms and medium tenure/earnings deceased workers.

3.2 Incumbent Workers Results

In our main specifications, which we discuss below, we focus on incumbent workers employed at the firm at the time of the (actual or placebo) death and who remain at the firm in the following years (incumbent stayers). We also study the effects on the

overall set of incumbent workers, including stayers and movers, in Appendix Table A.8. There, we also analyze the probability of switching employers, finding negative but no statistically significant effects.

Earnings and hours. We show effects on total annual earnings in Figure 3 and in column (1) of Table 3.² Relative to a control mean of 35,000 EUR we detect a statistically significant increase between EUR 350 and EUR 600 as a consequence of the death of a coworker, thus corresponding to an increase between 1% and 1.7% of total annual earnings. We further draw on the survey data described in Section 2.2 to dissect the earnings result into overtime and bonus earnings. First, table in column (2), we find an increase of earnings between 450 and 750 EUR in the event year and the subsequent two years, thereby matching the effects from the administrative data sample reported in column (1). We find a slightly smaller effect on earnings for the survey sample in the third post-event year of around 330 EUR. In table A.7 we turn to overtime earnings in column (4), we find positive and statistically significant effects between 35 and 85 EUR, suggesting that some—but not all—of the effect on overall earnings is driven by overtime pay. We next turn to bonus earnings in column (5) and find positive effects between 50 and 200 EUR that are statistically significant in the two years after the event. Summing up overtime earnings and bonus earnings, we find a statistically significant increase of 300 EUR, which correspond to an increase of 17%.

Hourly wages increase by 3.3% one year after the sudden departure, but the increase is short-lived and might be related to the mismeasurement of hours worked in administrative data.

Overtime hours. We next turn to the analysis of work hours in Figure A.3 and column (4) of Table 3.³ Here, we find a statistically significant increase in the year of the coworker death of about 21 hours, relative to a control mean of 1,580 hours. In the following years, we do not detect statistically significant effects on hours, suggesting that the incumbent coworker hours response is short-lived. We further check and reject in column (3) of Table A.7 that the hours response is driven by overtime as the point estimates are close to zero and cannot account for the spike in work hours during the event year.

Additional Specification Table A.8 reports results for all incumbent coworkers (stayers and switchers). The probability to change employers is not altered. The effects on

²We also analyze the effect on firms' overall wage bill in Appendix Table C.

³We also report the effects on hours worked based on the administrative data in column (3).

total earnings, total hours worked and hourly earnings are quantitatively similar. Table A.9 reports results for models that are not weighted by the inverse of the employer size, contrary to the main specification. In the unweighted specification larger firms receive more weight. We find that total earnings of coworkers increase but the magnitude of the effect is twice smaller, suggesting that small firms adjust more than large firms.

Summary. This section documents the effects of an unexpected worker death on firm and incumbent coworker outcomes. We use the pre-period coefficients to assess the plausibility of the common trends assumption and find flat pre-trends across all specifications. We find temporary negative effects on firms' employment and longer-lasting effect on firms' value added and profits. We also find positive and persistent effects on incumbent earnings and short-run increases in hours worked during the event year.

4 Theoretical Framework

This section describes a model of the labor market in order to organize and qualitatively match the empirical patterns we document in Section 3.

4.1 Setting

We draw on a search and matching model. These models are centered around an equation summarizing the equality of costs and benefits of creating new jobs. Hence, hiring is a form of investment, and the job creation equation is usually interpreted as the hiring costs that firms face (see Petrosky-Nadeau and Wasmer (2017)). In this section, we modify the textbook model to incorporate adjustment *within firms* and study how the value of a job change. We examine the implications first for labor supply estimates and hiring costs. The environment that workers and firms face is as follows. Time is continuous. The labor market features a continuum measure of workers and (active) firms. Both economic agents have an infinite life span and discount the future at rate $r \geq 0$.

Firms. Firms use one type of labor. The flow output Y is given by:

$$Y = A(N \cdot h)^\alpha, \alpha \in (0,1), \quad (3)$$

where N stands for the number of workers, h for the number of hours per worker, and $A > 0$ is a constant parameter representing the firm's productivity. As the firm's

production technology features strict concavity, worker productivity decreases with employment.

Labor market frictions imply that posted jobs are not instantaneously filled. Vacancies are filled at the exogenous rate $m > 0$. The filling rate of firms does not depend on the aggregate labor markets or the firm's characteristics. To post $V \geq 0$ vacancies, firms pay a strictly increasing, strictly convex, continuously differentiable cost $C(V)$ that satisfies $C(0) \geq 0$ and the Inada conditions $\lim_{V \rightarrow 0^+} C'(V) = 0$, $\lim_{V \rightarrow \infty} C'(V) = \infty$. Jobs are potentially infinite, but some workers leave their jobs randomly at exogenous Poisson rate $q > 0$. Workers do not search while employed. No on-the-job-search implies that this framework does not predict job-to-job quits.

We denote by $\Pi(A, N)$ the **value of the firm**, which satisfies the Hamiton-Jacobi-Bellman equation:

$$\begin{aligned} r\Pi(A, N) = & A [Nh(A, N)]^\alpha - Nw(A, N) - qN\Pi_N(A, N) \\ & + \max_V [-C(V) + mV\Pi_N(A, N)] + \dot{\Pi}(A, N), \end{aligned} \quad (4)$$

where Π_N denotes the derivative of Π with respect to its argument N and $\dot{\Pi}$ stands for the time derivative of Π .

Workers. The preferences of workers over hours worked h and consumption c are summarized as:

$$u(c, h) = c - \phi(h), \quad (5)$$

where $\phi(h) = \frac{h^{1+\eta}}{1+\eta}$, $\eta \geq 0$. η represents the inverse of the labor supply elasticity to the hourly wage in standard models. Workers are assumed to be risk-neutral. Workers can be either employed or unemployed. The consumption flow of employed workers is equal to their wage, denoted by w . The consumption flow of unemployed workers, who work zero hours, is equal to their unemployment income b .

Wage setting. Labor contracts define wages and hours contingent on the productivity parameter A and on the number of employees N , denoted by $w(A, N)$ and $h(A, N)$. Importantly, our approach does not rely on specific assumptions on wage setting and can accommodate the two leading models of wage setting in frictional labor markets. First, it can accommodate wage or contract posting models (Burdett and Coles, 2003), in which employers post non-renegotiable labor contracts that either fulfill the participation constraint of workers or maximize the value of a job vacancy. Second, our approach can also accommodate bargaining models, in which contracts are bargained

over either continuously or by mutual agreement (see, e.g., Pissarides (2009); Postel-Vinay and Turon (2010)). In all cases, we assume that labor contracts stipulate hours of work that satisfy the productive efficiency condition, or to put it differently, which maximizes the value of the job surplus of each worker.

4.2 Predictions

We highlight the qualitative predictions that our theoretical framework provides. Two testable predictions emerge from our framework: change in hours worked and in hiring costs after an incumbent worker quits.

Hours. The productive efficiency condition implies that the marginal productivity of labor is equal to its marginal disutility, $\phi'(h)$, which yields the optimal number of hours of work of each worker:

$$h(N) = (\alpha A)^{\frac{1}{1+\eta-\alpha}} N^{\frac{\alpha-1}{1+\eta-\alpha}}. \quad (6)$$

The number of hours of work of each worker decreases when there are more workers in the firm because the marginal product of labor is decreasing ($\alpha < 1$). The model predictions is in line with empirical evidence. For example, Cooper et al. (2007) show that hours growth and employment growth vary at the same rate, co-vary negatively, and hours growth in a period positively correlates with employment growth in the next period.

The magnitude of the elasticity of hours of work to employment, $\varepsilon_{h/N} = -\frac{1-\alpha}{\eta+1-\alpha}$, increases with the elasticity of labor supply ($1/\eta$) because workers whose labor supply is more elastic are more inclined to adjust their hours of work when their marginal productivity is changing.

Wages. The predictions about wages of these different models of labor contracts are more diverse. In bargaining models in which wages are continuously renegotiated, there is a unique wage function $w(A, N)$ which increases with the marginal productivity of labor. In other models, in which contracts are renegotiated by mutual agreement or posted by employers, there are multiple possible wage functions.

The job value. We deduce the job value, equivalently the hiring costs, in two step.

From the firm's optimization behavior (4) we deduce the **job value**:

$$\Pi_N(A, N) = \frac{C'(V)}{m} \quad (7)$$

The optimal number of vacancies implies that the marginal value of a filled job equals the hiring costs. Hiring costs depends on the marginal cost of posting a vacancy $C'(V)$ times the per-unit-of-time probability to fill vacant jobs ($1/m$). m is exogenous to aggregate labor market conditions or independent of the characteristics of the firm that posts the job.

Using equation (4) which defines the value of the firm, we can get the value of the marginal job to the firm:

$$r\Pi_N(A, N) = \alpha A [Nh(A, N)]^{\alpha-1} [h(A, N) + Nh_N(A, N)] - w(A, N) - Nw_N(A, N) + [mV(A, N) - qN]\Pi_{NN}(A, N) - q\Pi_N(A, N) + \dot{\Pi}_N(A, N). \quad (8)$$

First, in a steady state, firms do not grow or shrink. Hence $mV(N) = qN$, and the value of the marginal job does not change over time, hence $\dot{\Pi}_N(A, N) = 0$. Second, the term $A [Nh(A, N)]^{\alpha-1}$ is equal to $y = Y / Nh$, the ratio of output to hours worked. Finally, plugging the optimal number of vacancies equation (7) into the steady state value of a marginal job (8) :

$$(r + q) \frac{C'(V)}{m} = \alpha y h (1 + \varepsilon_{h/N}) - w (1 + \varepsilon_{w/N}) \quad (9)$$

where $\varepsilon_{h/N}$ denotes the elasticity of work hours to employment and $\varepsilon_{w/N}$ stands for the elasticity of wages to employment.

The right-hand side of this equation represents the undiscounted flow benefits that an employer gains from an additional worker. It represents the marginal cost of vacant jobs divided by the filling rate of vacant jobs. In the extreme case where a job can be filled instantaneously ($m \rightarrow \infty$), the cost to replace a worker goes to zero. The adjustment of hours worked, represented by ε_h , reduces the turnover cost because the time needed to recruit workers is less costly to the firms if it can adjust the hours of work of their employees. The adjustment of wages plays in the opposite direction if the wage increases with the firm size.

4.3 Discussion of Related Theoretical Framework

Our framework fits into the literature on labor search that incorporates firm size. In the standard framework (Pissarides (2000)), firms have one job. The standard framework is unsuited for the analysis of replacement costs and generally for the joint analysis of firm dynamics and labor market outcomes. As in Elsby et al. (2021) and Mercan and Schoefer (2020), we build a model to estimate the cost of replacing an employee. In contrast to those contributions, we highlight the adjustment of hours within the firms

to assess the cost of quitting. On wage determination, we follow Cooper et al. (2007) that specify long-term contracts that depends on the size and productivity of the firms. This approach does not tie the wage determination to elusive (to the econometrician) parameters.

In contrast to existing models, our framework offers a measure of labor turnover costs at the middle ground between the “structural” and “reduced-form” approach as advocated in Chetty (2009). Indeed, structural parameters that relate to the measure of labor turnover costs (e.g., ε_h , ε_w , α) can be estimated using transparent and credible identification assumptions. Besides, we are the first to bring the standard “job creation equation” of the DMP model to the data. In his survey on recruitment costs, Manning (2011) presents the DMP model as a natural framework to measure hiring costs but does not present evidence based on this model.

4.4 Identification of Structural Parameters

This section describes how we use exogenous worker separations to identify core parameters of labor market models. Drawing on the model in Section 4 and exogenous worker separations, we identify: α , the marginal product of labor and η , is the inverse of the Frisch labor supply elasticity and the marginal value of a filled job, Π_N (equal to the labor turnover costs). Concretely, we leverage exogenous worker separations as exogenous instruments on the labor inputs of firms.

Instrument. The instrument that we leverage to deal with the endogeneity of the labor input is the inverse of the number of employees for treated firms, and zero otherwise. The instrument does depend on the number of employees as the magnitude of the treatment varies with firm size. In coworker-level specifications, the instrumental variable is not weighted by firm size, as the number of coworkers already scales those regressions as in the event study specification 2. We test other specifications, reported in table A.10, A.11, A.12 and A.13. First, we also include a set of dummies indicating the characteristics of the deceased worker. Concretely, we include quintiles of age for the deceased treated and quintiles of earnings for the deceased treated. Second, to select the set of IV variables, we use the machine learning post-selection IV method (Belloni et al., 2014) to limit specification searching or p-hacking. Third, we use the binary outcome as IV, not the inverse for treated firms, and zero otherwise.

Worker death is a valid instrument as it satisfies the two conditions to get consistent estimates in the linear IV framework. First, we can motivate the exclusion restriction for the instrumental variable as there is no direct link between the outcome of interests (output, hours worked, wage) and the type of worker separations we leverage:

unexpected worker death. Indeed, worker separations, triggered by workers or firms, are related to the outcome of interests. For instance, a drop in total sales is correlated with worker separations. This phenomenon is well-documented in the labor literature. E.g., Davis et al. (2006) document the link between firm growth rate and worker separations due to quits. In our setting, separations are unexpected (from the firm’s viewpoint) and unplanned (from the worker’s viewpoint). Indeed, our event studies analysis (section 3) shows that firms that *will be* treated do not evolve differently relative to comparison firms in previous years. In other words, we do not find evidence that firms anticipate or cause worker separation. Second, worker deaths directly impact the endogenous variable, the labor input of firms. Section 3 shows that labor input (measured as total hours or the number of employees) is negatively impacted after the sudden worker exit of a worker. The main specifications in the next section show that weak instrument is not an issue in our setting.

Overall, the labor supply shock that we leverage for identification satisfies the exclusion restriction and instrument relevance, the two key conditions to get consistent estimates in the IV framework.

Main specification. For estimates of the elasticity of output, hours worked and wage to labor input, detailed below, we transform variables as the log difference before and after the relative year of the event, k . Concretely, any variable X is differentiated over a 3 year window to represent its growth rate: $\frac{1}{3}\sum_{k=-3}^{-1}\ln X_k - \frac{1}{3}\sum_{k=0}^2\ln X_k$.

Marginal product of labor. We start by using the relation between output Y , and the total labor input number Nh implied by the production function. The production function (3) leads to the following estimating equation:

$$Y_{it} = a_0 + a_1Nh_{it} + a_2X_{it} + \varepsilon_{it}, \quad (10)$$

the index it designates firm i in time t ; Y_{it} represents (log difference of) total revenue. X_{it} is a vector of control variables, including sector fixed effects, year fixed effects, age of firms, log of intermediate input costs and the log of the capital stock. ε_{it} is an error term. Nh_{it} denotes the (log difference of) total number of hours of work in firm i at time t . a_1 corresponds to the exponent α on the labor input, the marginal product of labor. In our theoretical framework, we assume that $\alpha < 1$, hence a_1 measures the degree to which the production function exhibits decreasing return to scale. In contrast to a large literature in empirical Industrial Organization that relies on structural

assumptions ⁴, we simply use a linear IV to recover the marginal product of labor.

Elasticity of coworker hours. The labor contract determines the optimal number of hours (equation (6)):

$$h_{ijt} = b_0 + b_1 N_{it} + b_2 X_{ijt} + \zeta_{ijt}, \quad (11)$$

where h_{it} stands for the (log difference of) annual hours of work of co-worker j in firm i . X_{ijt} contains variables used in the estimation of the production function (10) and characteristics of the co-worker j . Control variables for co-worker j are; sexe, age, years of education, log annual days covered by an employment contract⁵ and occupation dummies. Note that the richness of individual characteristics render possible to quantify the variation of the change in firm size on individual hours worked, after controlling for worker heterogeneity in several dimensions. b_1 captures the estimated elasticity of *individual* hours worked with respect to firm size, ε_h in our theoretical model. The comparative static of the theoretical model shows that hours worked decreased with the size of the firm, $b_1 < 0$. Cooper et al. (2007) measure the correlation of the growth rate (log first difference) of hours per worker and the growth rate of employment at the establishment level at the quarterly level. They find a negative correlation of -.29.⁶

Elasticity of coworker wages. Using the wage contract $w(A, N)$ that depends on firm size and productivity:

$$w_{ijt} = c_0 + c_1 N_{it} + c_2 X_{ijt} + \zeta_{ijt}, \quad (12)$$

where w_{ijt} stands for the compensation of co-worker j in firm i in year t . The set of control variables at the firm level and coworker level are similar to the estimates of hours worked. We use two definition of ‘compensation’. First, we use total annual earnings. Second, we use hourly earnings ($\frac{\text{Earnings}}{\text{Hours Worked}}$). The theoretical prediction does not provide guidance on the sign of c_1 . However, many frameworks seek to explain the relationship between wages and firm size. Especially, the monopsony literature uses a similar specification to recover the elasticity of labor supply to the individual firm and find estimates of c_1 between 0 and 0.5 (see Sokolova and Sorensen (2018)).⁷

⁴For instance, investments or intermediate inputs can be used to deal with the simultaneity problem (unobserved productivity is known to the firm when it decides the amount of inputs). Our firm panel is balanced, hence endogenous exit is not an issue.

⁵Days covered by an employment contract are a proxy for the number of days worked.

⁶On Danish firms, Trapeznikova (2017) find quantitatively similar negative correlation.

⁷In a standard monopsony model, the elasticity of firm size with respect to firm wage (the elasticity of labor supply to the firm) quantifies the implied degree of firms’ wage-setting power.

4.5 Structural Analysis: Empirical Results

We find that the elasticity of output with respect to labor in our data is 0.60, we obtain an intensive labor supply elasticity close to 0.30.

4.6 Elasticity of Output With Respect to Labor

Table 4 reports estimates of output to labor input, $\varepsilon_{Y/Nh}$, for different specifications. We measure total output as total sales, and labor input is total hours worked. All estimates of marginal product of labor are below one, validating the functional form of the production technology used in our theoretical framework ($Y = (Nh)^\alpha$). We control for year fixed effects, relative time fixed effects, and employer characteristics (age of firms, log of intermediate inputs, log of the capital stock).

The OLS estimate of the marginal product of labor is $\varepsilon_{Y/Nh} = 0.34$ (column 1). Even with control variables, the literature argues that unobserved inputs such as managerial ability bias input elasticities. The sign of the bias is unclear as more productive firms could buy more or fewer inputs. We implement an instrumental variable strategy to control for endogeneity. Our instrumental variables stem from the exogenous belonging of the treated and control group. The F-statistics for weak identification is above the conventional weak instrument threshold of 10 (Andrews, Stock and Sun, 2019). Including IV variables, we find a marginal product of labor equal to 0.61 (column 2), compared to an OLS point estimate of 0.34. The negative bias of OLS indicates a negative link between unobserved heterogeneity and labor input. After controlling for unobserved productivity using panel data, Bagger et al. (2014) also find a negative bias of OLS.

Additional specification. Additional specifications reported in table A.10 and table A.11 strengthen the confidence in the robustness of our estimates. The machine learning IV method (Belloni et al., 2014) produces a similar estimate ($\varepsilon_{Y/Nh} = 0.55$, column 3). The wage bill, instead of total hours worked to control for labor quality produces similar input elasticity (table A.10). Estimates in level rather than log differences (table A.11) produce similar estimates. Labor input elasticity in the range of 0.3 and 0.6 are consistent with other studies that estimate production function on small firms (e.g., Giupponi and Landais (2020)).

4.7 Labor supply elasticity.

Table 5 reports estimates of the elasticity of individual hours worked to employer size. In contrast to the marginal product of labor estimates, we estimate the elasticity of hours worked to employer size at the *coworker-level* to control for worker characteristics. We control for year-fixed effects, relative year-fixed effects, employer characteristics (age of firms, log of intermediate input costs, log of the capital stock), and coworker characteristics (occupation fixed effects, sex, age, education, log annual days covered by an employment contract).

We find that the elasticity is negative and statistically significant. This result is consistent with the comparative static properties of the theoretical framework (section 4). In column 1 of table 5 we report the OLS estimate. We find that a 10 % increase in firm size (number of employees) decreases the hours worked by coworkers by about 0.29%. After correcting for endogeneity, the estimate goes down to -2.7%.

Combining this elasticity with the elasticity of labor input elasticity, we can estimate η^{-1} . The estimand of the intensive margin elasticity of hours worked (the inverse of the disutility of working longer hours) is:

$$\text{Labor Supply Elasticity} = \eta^{-1} = \frac{\varepsilon_{h/N}}{(1 + \varepsilon_{h/N})(\varepsilon_{Y/Nh} - 1)}$$

As $\hat{\varepsilon}_{h/N}$ and $\hat{\varepsilon}_{Y/Nh}$ are estimators of the estimands $\varepsilon_{h/N}$ and $\varepsilon_{Y/Nh}$, the obvious estimator for η^{-1} would be to directly plug those estimators. But it would result in a biased estimator. Indeed, Jensen's inequality implies that $E[1/\hat{\theta}_2] \geq 1/E[\hat{\theta}_2]$.

Instead we obtain estimates $\varepsilon_{h/N}$ and $\varepsilon_{Y/Nh}$ by bootstrap. The mean and standard errors across all bootstrap samples are reported in table 6. The bootstrap standard errors are obtained as the sample standard deviation of the labor supply estimate across all bootstrap samples.

The estimates of the elasticity of labor supply elasticity are close to recent quasi-experimental evidence (Chetty et al., 2011).

Additional specification. We report additional results on the relationship between employer size and the hours worked by coworkers. All additional specifications, reported in A.12 and A.13 qualitatively show that hours worked are negatively correlated with employer size.

4.8 Turnover Costs

Table 7 reports the decomposition of the turnover costs. The structural model provides a framework to decompose the turnover costs (see equation 9). The turnover costs depend on the marginal gain of the additional employee and the adjustment of hours and wages of coworkers. To obtain standard errors on turnover costs, we bootstrap b times the original dataset.

We proceed as follows:

1. Estimate $\alpha_{(b)}$, $\varepsilon_{(b)h/N}$ and $\varepsilon_{(b)w/N}$ for every bootstrap sample b . Averaging over the samples, point estimates of the structural parameters are; .62 (SE .17), -.44 (SE .17), and -.82 (SE .23) for α , $\varepsilon_{h/N}$ and $\varepsilon_{w/N}$ respectively.
2. Compute the mean productivity ($\bar{y}_{(b)}$), mean hours worked ($\bar{h}_{(b)}$), and mean salary ($\bar{w}_{(b)}$), using annual Firm-Level data for every bootstrap sample b .⁸ Averaging over the samples, 30 EUR is the hourly productivity, 1455 is the annual hours worked and 30,640 EUR is the annual salary.

Which we plug into the turnover cost equation 9:

$$(r + q) \frac{C'(V)}{m} = \underbrace{\alpha_b \bar{y}_{(b)} \bar{h}_{(b)} - \bar{w}_{(b)}}_{(1) \text{ Marginal Job}} + \underbrace{\alpha_{(b)} \bar{y}_{(b)} \bar{h}_{(b)} \varepsilon_{(b)h/N}}_{(2) \text{ Hours Adjustment}} - \underbrace{\bar{w}_{(b)} \varepsilon_{(b)w/N}}_{(3) \text{ Wage Adjustment}} \quad (13)$$

- (1) $\alpha_b \bar{y}_{(b)} \bar{h}_{(b)} - \bar{w}_{(b)}$: annual value added of an employee factoring in the diminishing return, minus salary of a typical employee. In our sample, the net value of an additional employee is negative, as the benefit is evaluated at 27,980 EUR ($\alpha_b \bar{y}_{(b)} \bar{h}_{(b)}$) and the cost at 30,640 EUR. In macroeconomics, the gap between value added of an employee and its costs is a measure of labor misallocation.
- (2) $\alpha_{(b)} \bar{y}_{(b)} \bar{h}_{(b)}$ 0: The adjustment of hours decreases the value of marginal jobs because when the size of firms increases, the marginal productivity of labor drops, which induces workers to work less hours.
- (3) $-\bar{w}_{(b)} \varepsilon_{(b)w/N} > 0$: empirically $w'(A, N) < 0$ which implies that the adjustment of wages increases the value of marginal job.

To benchmark our results, we directly estimate the turnover costs or the value of a marginal job for a firm. The estimator that we use to produce direct estimates of

⁸ y_{it} : value added over total hours worked. h_{it} : total hours worked over average number of employees. w_{it} : wage bill over the average number of employees. Firm-Level data are used for treated and comparison firms, before the year of the event.

the turnover cost is: $\hat{\Pi} = \hat{\beta}_0^T + \hat{\beta}_1^T + \hat{\beta}_2^T$, where $\hat{\beta}_{0,1,2}^T$ refers to the estimates from the difference-in-differences model (1). Using this estimator, the cumulative negative effect on profit of losing one employee amounts to 41,000 EUR (SE: 1003 EUR).

5 Discussion of Related Research

Here, we discuss our evidence vis-à-vis the existing literature and also compare and contrast our estimates.

The replacement cost. Replacing employees that leave voluntarily their jobs is the most common reason for a job opening. Mercan and Schoefer (2020) estimates that 56% of job openings in Germany aims at replacing a worker.

Our work provides direct evidence of the cost bear by firms after unexpected worker exits. This work sheds light on the adjustment process in labor markets from the viewpoint of firms, which is less documented than from the perspective of workers. In his literature review, Manning (2011) noted the lack and diversity of hiring cost estimates. The scarcity of hiring cost estimates is an important gap in the literature for several reasons. First, theories of unemployment rely on the idea that labor turnover is costly. Second, voluntary separations are the predominant cause of job separations (Davis et al., 2006). Third, recent work emphasizes the role of replacement hires as essential to explain employment fluctuations (Mercan and Schoefer, 2020; Elsby et al., 2021)

In one of the earliest such studies, Oi (1962) documented hiring costs of about 5% of the wage bill, in line with most papers overview in Manning (2011). The bulk of hiring costs come *after* hiring takes place. Using representative Swiss establishment-level survey data, Muehleemann and Leiser (2018) decompose the total costs to replace workers with a vocational degree. They find that the formal training, the low initial productivity of a new hire, and the time required by other workers to instruct a new hire represent 79% of the firm's hiring costs.⁹ Beyond survey-based estimates, in a recent case study paper, Kuhn and Yu (2021) estimate the effect of quits of retail salespeople on the establishment's productivity for one large firm in China. Quits lead to productivity losses after the worker gives notice before she departs, after she leaves, and after a new worker starts.

In contrast to Kuhn and Yu (2021), our estimates of turnover costs are for a wide variety of occupations using administrative datasets, and we combine quasi-experimental

⁹Boushey and Glynn (2012) survey 31 case studies and conclude that firms pay about one-fifth of an employee's annual salary to replace that employee. Villena-Roldan (2012) report that firms in the National Employer Survey 1997 (NES97) declare to spend 2.5% of their total labor cost in recruiting activities.

evidence with a structural decomposition of the labor turnover costs. Indeed, we provide a novel methodology for estimating the marginal value of a filled job or the costs of turnover to the firm. We make use of the job creation equation of a matching model (Pissarides, 2000), which equalizes the marginal hiring costs.

The elasticity of labor supply. Our evidence complements an existing body of literature, surveyed in Chetty et al. (2013), aimed at estimating intensive- and extensive-margin labor supply elasticities. The authors present a meta-analysis of micro estimates with a mean value of 0.3 at the intensive margin. Our approach is different as we do not use exogenous change in the wage to estimate the responsiveness of wages on hours worked. In contrast, we use as an instrumental variable the loss of one worker for *some* firms in our sample to estimate the elasticity of average hours worked of incumbent workers on a firm's employment. Using the latter elasticity and the output elasticity on employment, we back up the intensive elasticity of labor supply.

A novelty of our approach is that the elasticity of labor supply depends on firms' structural parameters. Pencavel (2016) asks whether the association between hours of work and wages reflects the preferences of workers or the preferences of employers. Battisti et al. (2021) also highlight the role of the assumption of firms' technologies on the estimates of labor supply elasticity. Their key point is that complementarities between workers in a firm attenuate working time adjustments to individual-specific variation in work incentives. In this paper, we highlight the role of the adjustment of hours within firms and the concavity of the production technology to recover labor supply elasticity.

6 Conclusion

We analyze shock to firm-specific labor supply due to unexpected worker deaths of workers, and we show that firms face high turnover costs. We show this by finding a negative effect on a firm's amount produced and that such an event affects firms' labor demand for the remaining workers. Our analyses of more than 1,700 events reveal that losing an employee lowers profits in the subsequent four years by roughly one year of the total labor costs for an average employee. We interpret our results through the lens of a search and matching model that explain turnover costs depending on workers' preferences and the firm's underlying production technology.

A key difference in our study relative to most estimates from the literature is that we rely on natural experiments and detailed data on firm production. Up until now, the literature has used employer surveys or data on workers to back out replacement

costs (see, e.g., Manning 2011; Jäger and Hening 2022). Data on the firm's production allows us to shed new light on replacement costs that complement existing literature.

To circumvent the endogeneity of worker exits, we limit our analysis to the effects of worker exits due to death. It seems plausible that our estimates could be used to understand more generally the effect of separations initiated by employees. Quits are an important labor market phenomenon, as quits and not layoffs trigger most job transitions. Nonetheless, we recognize that unexpected exits that we analyze may only generalize to other exits partially. A credible research design for non-death worker exits would complement our study. Also, our results represent replacement costs for small firms in Denmark. Even similar research designs can lead to very different results depending on the specificities of the labor markets under study (Bertheau et al., 2022). Therefore, similar analyses in different countries would help to benchmark our results.

Finally, our findings point towards additional work on better understanding the matching process of workers to firms. Using a Randomized Control Trial, Algan et al. (2022) shows that firms to hire has a positive effect on employment and revenue. This result suggests that targeted public policy focusing on the employer side can have important economic benefits. An additional route would be to use large-scale surveys to document employers' perceptions, attitudes, and reasoning to understand better the mechanisms generating our results.

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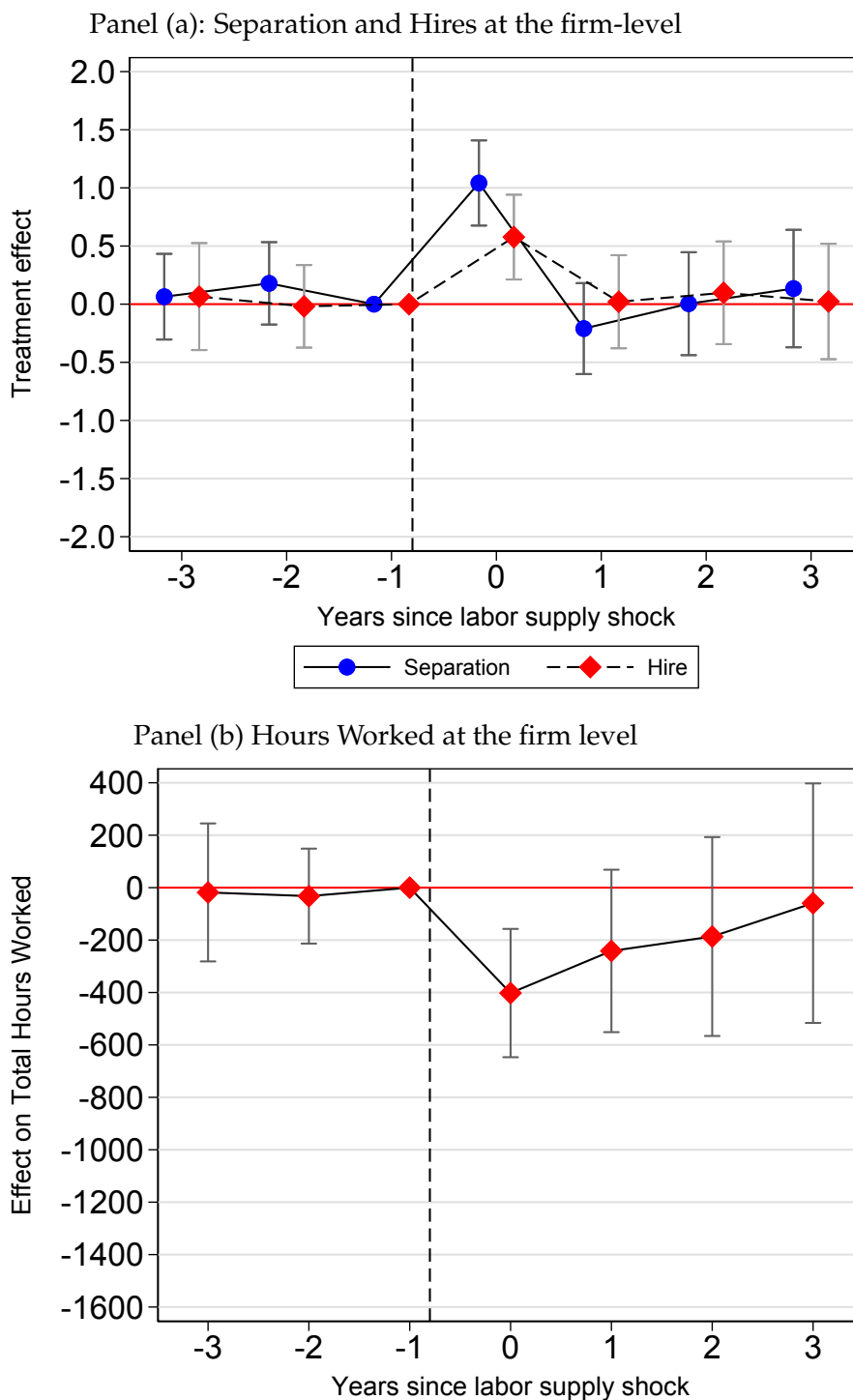
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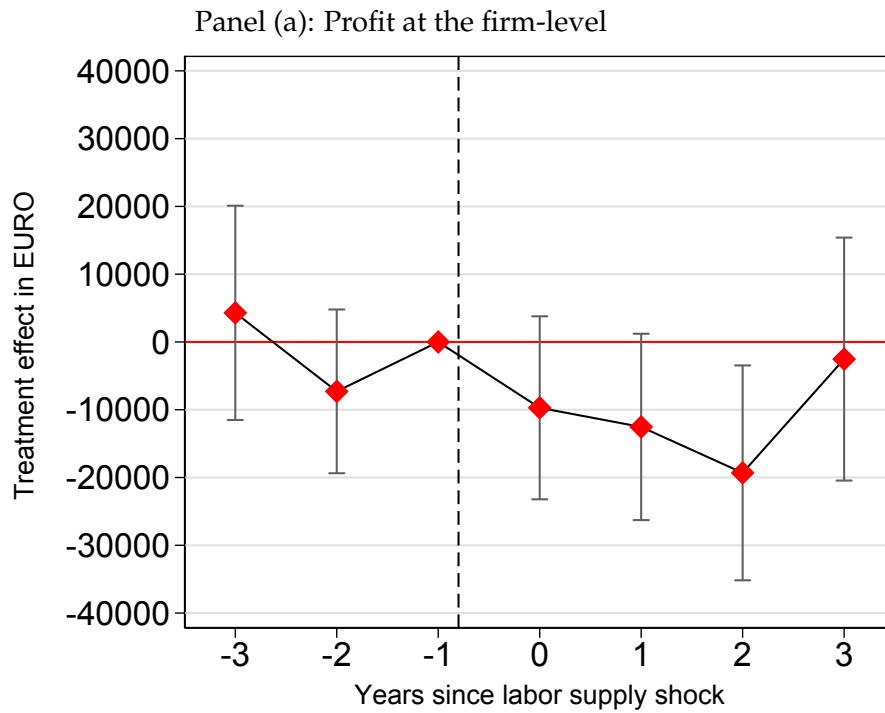
Results: Figures

Figure 1: The Effect of Worker Death on Separation, Hires and Total Hours

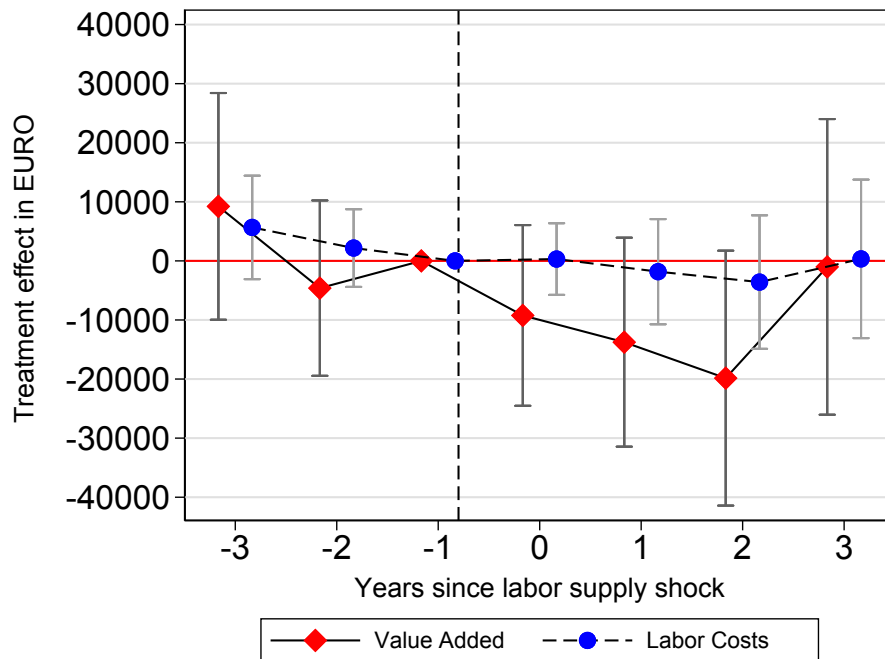


Note: The figure shows event study estimates from equation (1). Estimates are relative to $k^* - 1$, where k^* is the worker death year. The outcome variable measures the number of separations and the number of hires in Panel (a). Panel (b) reports the number of total hours worked at a firm at the end of the year. The point estimates and standard errors are reported in table 2. Section 3.1 discusses the results.

Figure 2: The Effect of Worker Death on Profits

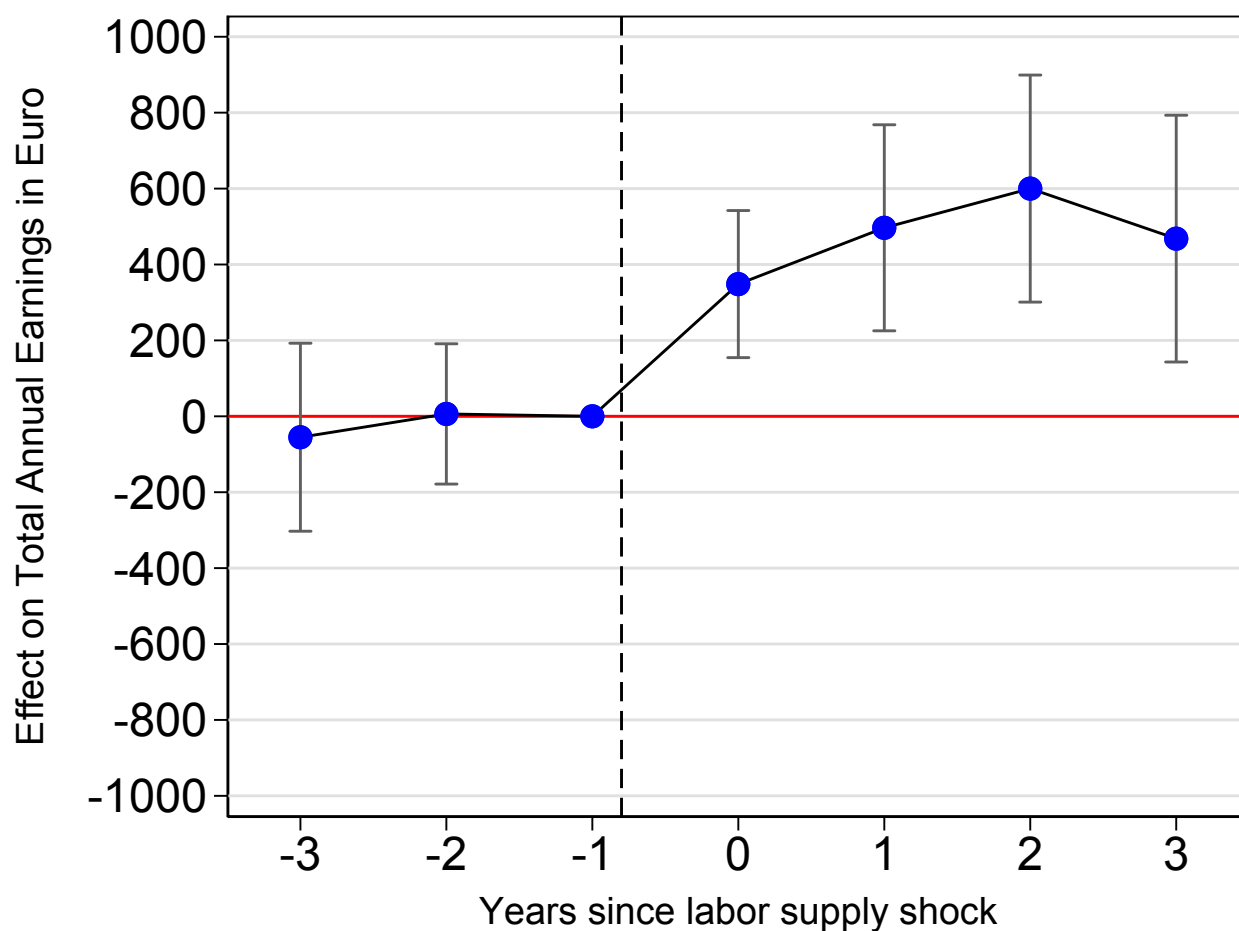


Panel (b) Decomposition of profits into value added and labor costs



Note: The figure shows event study estimates from equation (1). Estimates are relative to $k^* - 1$, where k^* is the worker death year. Panel (a) reports estimates for the profit of the firm. Panel (b) decomposes the profit into its components: value-added and labor costs. The point estimates and standard errors are reported in table 2. Section 3.1 discusses the results.

Figure 3: The Effect of Worker Death on on Incumbent Worker Earnings



Note: The figure shows event study estimates from equation (2). Estimates are relative to $k^* - 1$, where k^* is the worker death year. The outcome variable is annual earnings for incumbent workers who remain at the firm in the following years (incumbent stayers). Each observation is weighted inversely by the number of incumbent workers at a firm. The point estimates and standard errors are reported in table 3, column 1.

Tables

Table 1: Summary Statistics on Treated and Comparison Groups

Panel (a): Treated and Comparison Firms					
	Comparison Firms		Treated Firms		p-value
	Mean	sd	Mean	sd	
# Employees	13.12	6.79	13.12	6.79	1.00
# Total hours	19,228	10,569	19,210	10,701	0.96
# Hires	7.32	8.64	7.18	7.91	0.63
# Separations	7.19	8.58	7.02	7.97	0.55
Value Added (VA)	669,603	468,321	661,150	464,979	0.61
Labor Cost (LC)	447,891	282,748	443,097	277,448	0.63
Profit (VA-LC)	221,469	298,997	217,543	301,656	0.71
Capital stock	553,020	1,790,232	582,279	2,205	0.68

Panel (b): Treated and Comparison Incumbent Workers					
	Comparison Worker		Treated Worker		p-value
	Mean	sd	Mean	sd	
Age	44.39	12	44.03	13	0.06*
Education (in years)	12.63	2.6	12.56	2.7	0.20
Men	0.73	.44	0.72	.45	0.43
Annual Labor Earnings	35,437	14542	34,925	14387	0.17
Annual Hours worked	1,548	320	1,544	334	0.51
Wage	22.85	8.8	22.65	9.3	0.43

Note: The table reports summary statistics for the treated and comparison group one year before the event. The table illustrates that treated and comparison firms are similar. ‘sd’ refers to standard deviation and ‘p-value’ reports the p-value of the t-test of the statistical significance of the treatment status in mean outcomes. The sample contains 1,728 treated and comparison firms. Section 2.3 defines the treated and comparison group and the sample restriction. Panel (b) shows the sample at the worker level. The sample contains 10,385 treated and 11,269 comparison coworkers.

Table 2: Effect of Worker Death on Firms

Lead /Lag	# Employees	# Separations	# Hires	# Total Hours Worked	Value Added €	Profit €	Value Added (Accounting definition, €)	Profit
-3	0.0920 (0.121)	0.0642 (0.224)	0.0654 (0.280)	-18.31 (160.0)	9676.2 (11122.1)	6211.4 (10538.1)	9225.8 (11661.9)	4295.4 (9608.0)
-2	-0.0475 (0.0978)	0.179 (0.215)	-0.0185 (0.216)	-32.34 (110.1)	-1056.1 (9642.8)	-2377.5 (9608.1)	-4608.8 (9020.3)	-7288.7 (7342.5)
-1	0 (.)	0 (.)	0 (.)	0 (.)	0 (.)	0 (.)	0 (.)	0 (.)
0	-0.331*** (0.114)	1.042*** (0.223)	0.577*** (0.222)	-402.2*** (148.8)	-3524.0 (10634.4)	-7369.2 (10341.1)	-9230.5 (9295.0)	-9716.4 (8206.2)
1	-0.281** (0.141)	-0.210 (0.238)	0.0208 (0.243)	-241.5 (188.3)	-12113.7 (11929.2)	-8459.3 (11139.9)	-13756.4 (10748.5)	-12536.6 (8355.2)
2	-0.215 (0.170)	0.00405 (0.269)	0.0978 (0.268)	-186.5 (230.6)	-11295.2 (13702.1)	-7196.4 (12156.3)	-19844.7 (13113.9)	-19314.3** (9633.7)
3	-0.0174 (0.196)	0.134 (0.307)	0.0231 (0.302)	-59.30 (277.7)	-10363.0 (15581.6)	-12362.3 (12895.4)	-1016.8 (15207.8)	-2527.0 (10893.9)
Mean	13.12	7.19	7.32	19,228	591,232	179,342	669,603	221,469
N	24192	24192	24192	24192	24192	24192	21531	21531

Note: The table shows event study estimates from equation (1). Estimates are relative to $k^* - 1$, where k^* is the worker death year. Standard errors clustered at the firm level are reported in parentheses. 'Mean' reports the control group means for the pre-event year $k = -1$. Stars denote statistical significance: *** p0.01, ** p0.05, * p0.1. Appendix B details variables construction.

Table 3: Effect of Worker Death on Coworkers

Lead Lag	Earnings	<i>Earnings</i> ^S	Hours Worked	Hours <i>Worked</i> ^S	Wage	<i>Wage</i> ^S
-3	-55.16 (150.6)	282 (395)	-7.410 (5.632)	19.05 (14.35)	-0.108 (0.165)	-0.214 (0.190)
-2	6.269 (112.3)	-102 (328)	1.808 (12.43)	19.02 (13.14)	-0.145 (0.104)	-0.364 ** (0.151)
-1	0 (.)	0 (.)	0 (.)	0 (.)	0 (.)	0 (.)
0	348.4* * * (117.8)	479.5** (224)	6.302 (4.530)	21.34** (8.79)	0.0555 (0.206)	-0.116 (0.145)
1	496.6* * * (165.0)	519 (341)	3.376 (12.40)	8.476 (12.37)	0.771* (0.440)	0.136 (0.167)
2	600.0* * * (181.8)	719 ** (337)	11.05* (5.997)	12.54 (11.28)	0.258 (0.213)	0.130 (0.186)
3	468.1** (197.6)	337 (322)	7.126 (6.162)	11.37 (11.83)	0.216 (0.240)	0.001 (0.201)
Mean	35,800	39,308	1,559	1,587	22.9	24.8
N	147,414	30,902	147,414	30,902	147,414	30,902

Note: The figure shows event study estimates from equation (2). Estimates are relative to $k^* - 1$, where k^* is the worker death year. Exponent S indicates that data are from the pay component data (LONN). The sample includes incumbent workers that do not switch years (stayers). Each observation is weighted inversely by the number of incumbent workers. Standard errors clustered at the worker level are reported in parentheses. Stars denote statistical significance: *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$. Appendix B for information on variables construction. Table A.8 reports estimates for the entire set of incumbent workers (stayers and switchers).

Table 4: Output Elasticity to Labor Input

	OLS	IV
$a_1 = \alpha$.344*** (.019)	.614*** (.183)
Controls	✓	✓
F-test IV		16.1
N	3,456	3,456

Table 5: Hours Worked and Earnings Elasticity to Firm Size

	OLS	IV
$b_1 = \epsilon_{h/N}$	-.029** (.012)	-.267** (.130)
$c_1 = \epsilon_{w/N}$	-.007 (.012)	-.570*** (.162)
Controls	✓	✓
F-test IV		45.7
N	20,178	20,178

Note: Labor supply elasticity is estimated as

$$\frac{\epsilon_{h/N}}{(1 + \epsilon_{h/N})(\epsilon_{Y/Nh} - 1)}$$

Table 6: Bootstrap Estimates of labor supply

Parameters	Mean	SE
α	.62	0.17
$\epsilon_{h/N}$	-.44	0.17
Elasticity of Labor Supply	0.39	0.39

Table 7: Decomposition of Labor Turnover Costs

	Mean	SE	Share ($\bar{\Pi}'$)
(1) + (2) + (3):	10,089 EUR	6,954	1
(1) Marginal Job: $\alpha y_i h_i - w_i$	-2,659 EUR	7,907	
(2) Hours Worked Adjustment: $\alpha y_i h_i \epsilon_{h/N}$	-12,412 EUR	6,127	
(3) Earnings Adjustment: $-w_i \epsilon_{w/N}$	25,161 EUR	7,145	

Note: The Table reports the value of a marginal worker, following the decomposition 13.

Appendix

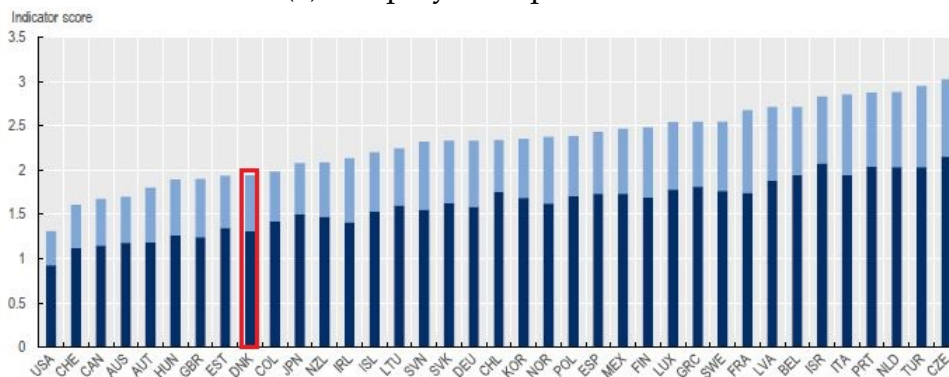
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A Additional Figures

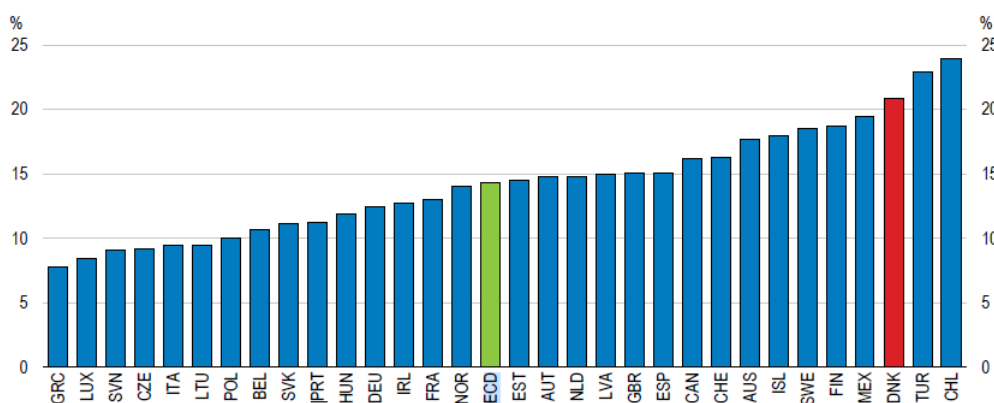
Figure A.1: Institutional Features of the Danish Labor Market

Panel (a): Employment protection is low



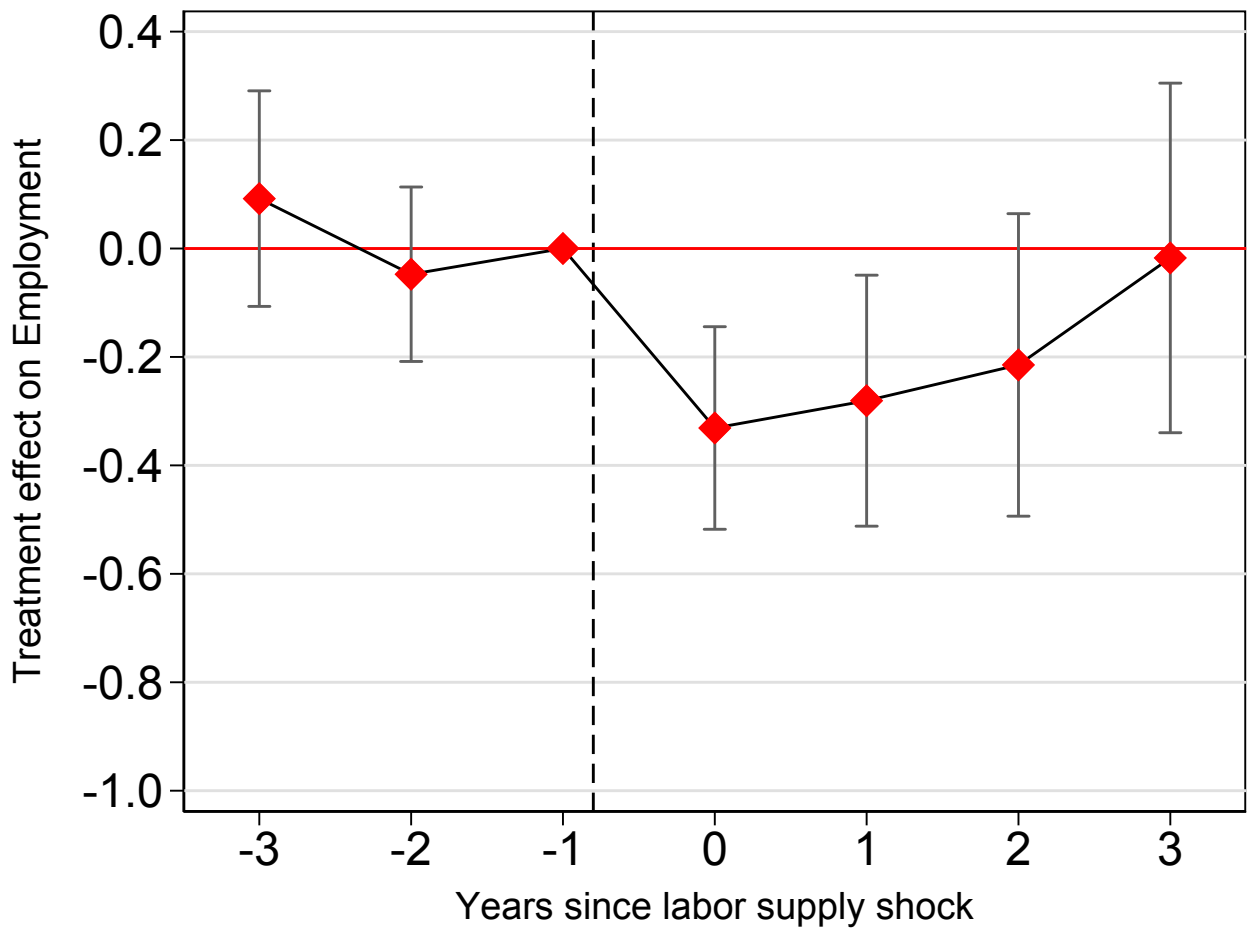
Panel (b): Job mobility is high

Job separation rate, 2017



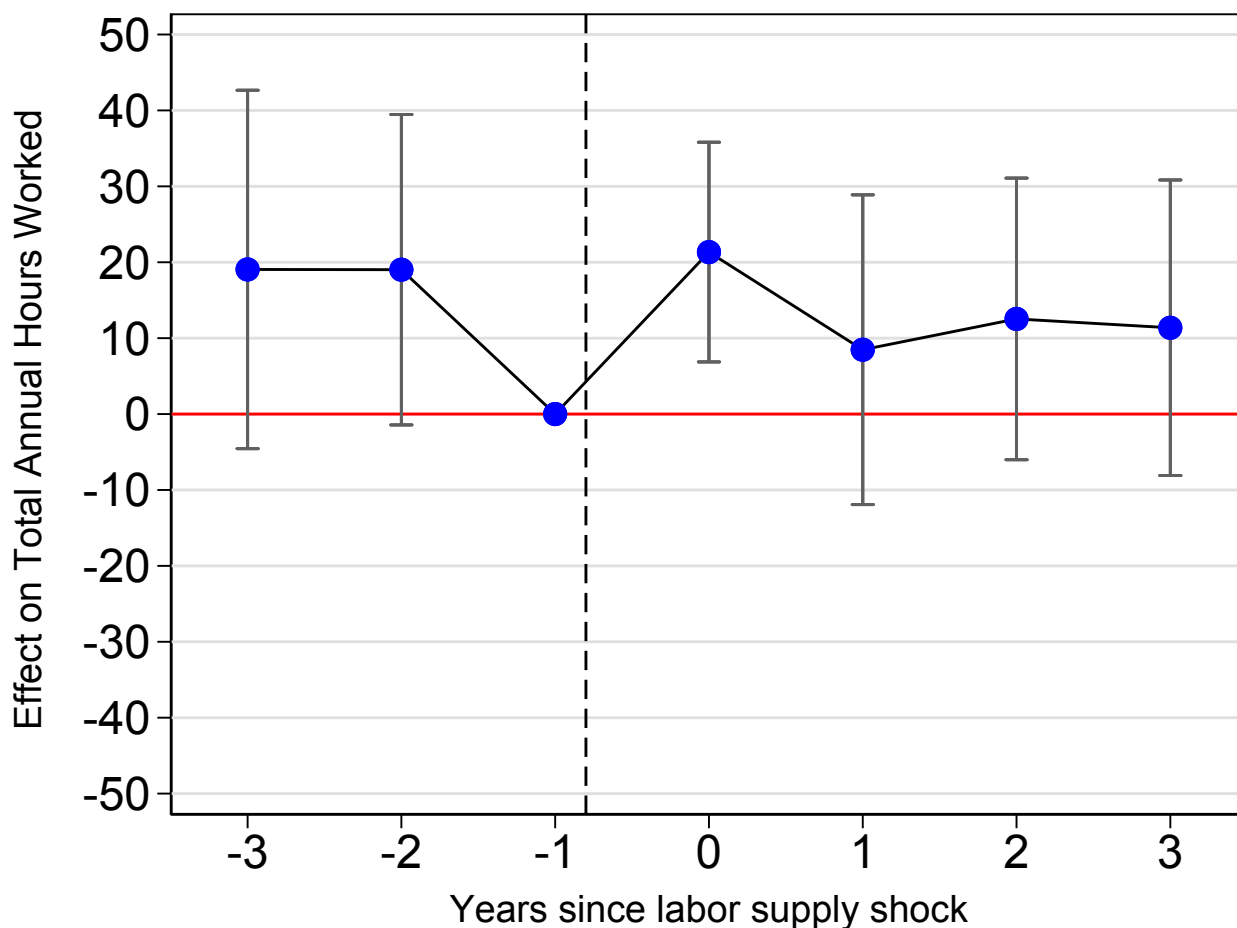
Note: The different panels report different features of the Danish labor market. This Figure illustrates that worker representation is high, employment protection is low, and that job mobility is high in Denmark. Source: OECD (2017, 2020). Section 2.1 in the main text describes the institutional context of the Danish labor market.

Figure A.2: Effect of Worker Death on Employment



Note: The figure shows event study estimates from equation (2). Estimates are relative to $k^* - 1$, where k^* is the worker death year. The outcome variable is the number of employees.

Figure A.3: Effect of Worker Death on Incumbent Worker Hours Worked



Note: The figure shows event study estimates from equation (2). Estimates are relative to $k^* - 1$, where k^* is the worker death year. The outcome variable is total hours worked at the end of the year. Observations are weighted inversely by the number of incumbent workers at a firm. Here, we report estimates for incumbent workers employed at the firm at the time of the (actual or placebo) death and who remain at the firm in the following years (incumbent stayers). The regression coefficients and standard errors are reported in table 3.

B Data Sources and Matching Procedure

Statistics Denmark provides the datasets used. For the analysis, we deflate all Danish Kroner variables to correspond to the 2000 CPI and convert them to EUR by applying a factor of 0.1339.

Data sources to construct the worker panel

1. Statistics Denmark does not provide a unified record of employment spells. Instead, we use the dataset used by Bertheau and Vejlin (2022). Labor Market History (CONESR,RAS,BFL) records from 1985 to 2013 all employment spells (jobs) at the yearly level. Each employment spell contains; firm identifier, earnings, hours worked, and start and end of employment spell. Data come from information sheets employers must send annually to the tax administration (SKAT) up to 2007 (CONS, RAS). Since 2008 (BFL), employers must report at the monthly frequency; total earnings, total paid hours, and occupations. Before 2008, hours worked were estimated using a step function corresponding to the employer's contributions to a mandatory pension scheme (ATP, *Livslang Pension*). The pension contribution is null for less than 9 hours worked per week. From 9 to 18, employers contribute one-third. From 18 to 27 hours, employers contribute two-thirds. For more than 27 hours, the contribution is total. Hence, hours worked for a given number of employees vary if some of the firm's workers switch between the 9-hour brackets. Statistics Denmark computes the range of possible hours worked that is consistent with collected weeks of pensions and length of employment spells. We compute gross labor earnings using the variable `lonb1b`.
2. Earnings Statistics (*Lønstatistikken*, LON). This dataset breaks apart earnings and hours worked for all firms (public and private sector) with more than 10 employees since 1997. Hours and earnings are divided into several components. Hours worked is decomposed as follow; baseline hours plus overtime hours minus hours of absence minus hours of holidays. Earnings are decomposed as follows; salary plus absence payments plus holidays payments plus pension payments plus personal benefits payments plus compensating payments plus irregular payments (e.g., bonuses).
3. We exploit other registers (IDAPALL)¹⁰ to retrieve the socio-demographic characteristics of workers. The number of years of education (`hfpria`), occupational groups (`pstill`), sex (`koen`), and age (`alder`) are retrieved from this dataset.

Data sources to construct the firm panel

1. The General Firm Statistics (FIGF, FIGT, FIRM) records financial statements (income and balance sheet statement), the age of firms, municipality code, and the value of export and

¹⁰This dataset is part of the ECONAU project database at Aarhus University.

import of firms from 1992. We report the definition of variables used by Statistics Denmark. Value added (vt , $gf-vtv$) includes revenues minus costs.¹¹ We define Capital as fixed assets (aa , $gf-aar$).

2. VAT data (MOMS, FIKS) is collected to comply with administrative rules on reporting the VAT tax. An issue to match with General Firm Statistics data is that a collection unit is a tax number, whereas in the General Firm Statistics, the unit is the legal number of firms. The reporting frequency is annual for large firms and monthly for small firms. We use total sales ($salg-ialt$) and total expenses ($koeb-ialt$).

Data sources to identify unexpected worker separations

1. Death Register (DODSAARS and DODSAARSG). The cause of death register records the date, cause, and location of deaths. Causes of death are classified using the International Classification of Diseases (ICD), which is a detailed coding of diseases.

In the event of employee death, the Danish law for salaried employees (*fundtionæloven*, Section 8) gives the right from one to three months of salary for the spouse or children depending on the tenure. *Fundtionæloven* does not apply to all occupations. We drop deaths where the location is reported at a workplace (c -handsted) and where workers are not covered by an employment contract at the time of the reported death (d -doedsdato).

2. Government Transfers (DREAM) record from 1991-2017 at the weekly frequency, all government transfers: e.g., efterløn (retirement pay), fleksydelse (flexible jobs benefits), sygedagpenge (sickness benefits) Aktivering (active labor market policies), orlov (leave benefits), VUS/SVU/SU (education grants) and unemployment benefits.
3. IDAP (IDA Person) contains demographics characteristics on the the Danish population. We exclude workers that die that are classified as employers, unemployed or early retirement receivers (Efterlønsmodtager) (PSTILL: 11, 50, 40) the year preceding the event.

¹¹Specifically: Revenue, Work done for own account and listed under assets, Other operating income, change in inventory goods consumption, subtracted by purchases of goods for resales, raw materials, energy, subcontracting work, expenses for rent, small inventories, temporary employment agencies, Long-term rental and operational leasing expenses, ordinary losses on debtors - Other external costs

C Additional Tables and Figures

Table A.1: Wage Systems in the Danish Labor Market

Year	1989	1991	1993	1995	1997	2000	2004	2007	2010	2012	2014	2017
Regulated pay	34	19	16	16	16	15	16	16	16	17	19	20
Decentralized pay	66	81	84	84	84	85	84	84	84	83	81	80

Note: Percentage of employees in the private sector covered by sectoral agreements that regulate pay via wage floors ("normallønssystemet") and sectoral agreements without or few wage floors. Source: Dahl et al. (2013) and Danish Employers' Federation (DA, 2018). The figure illustrates that most employees are covered by decentralized wage setting. Section 2.1 in the main text describes the institutional context of the Danish labor market.

Table A.2: Individual-Level Summary Statistics

Table 2.A: Treated and Comparison Incumbent Worker Summary Statistics

	Comparison Worker		Treated Worker		p-value
	Mean	sd	Mean	sd	
Survey Data					
Hours worked	1,545.48	357	1,520.06	352	0.13
Overtime Hours	25.46	111	19.50	77	0.25
Earnings	38,260	13842	38,086	13812	0.76
Overtime Earnings	363.07	1472	316.70	1312	0.53
Bonus Earnings	1,414	3426	1,349	3614	0.76
Wage	24.97	8.1	25.29	8.4	0.42

Table 2.B: Deceased Worker Summary Statistics

	Mean	Median
Age	54	55
Hours Worked	1263	1652
Experience	12	13
Tenure	5	4
Men	0.80	

Note: Summary statistics are measured one year before the event for incumbent workers that do not switch firms (Incumbent-Stayer). 'p-value' is the p-value of t-test of the significance of the treatment status on the mean outcome. The sample contains 10,385 treated and 11,269 comparison coworkers. In survey data, the sample contains 2841 treated and 2782 comparison coworkers. Table A.16 reports summary statistics measured on the last three years before the event for Incumbent-Stayers. Table A.17 reports summary statistics for the entire set of Incumbent workers (stayers and switchers). Appendix B details variables construction.

Table A.3: Effect of Worker Death on Firms: Impact on Profit

Lead / Lag	Total Revenue €	Intermediate Input Costs €	Labor Costs €	LaborCosts ^S €
-3	11366.0 (19615.5)	2794.8 (15315.4)	3166.9 (3530.9)	5665.4 (5320.9)
-2	-5472.0 (14298.2)	-2088.9 (11789.4)	1882.3 (2357.9)	2173.3 (3996.4)
-1	0 (.)	0 (.)	0 (.)	0 (.)
0	-14379.3 (17485.9)	-9845.4 (13236.7)	2851.2 (3098.1)	309.4 (3685.9)
1	-6201.0 (22706.4)	4935.1 (18209.1)	-3745.9 (4332.1)	-1825.4 (5409.1)
2	-1647.7 (27645.9)	11307.9 (21920.6)	-3282.2 (5521.1)	-3586.1 (6869.7)
3	15613.6 (32705.7)	27106.4 (25008.9)	-886.4 (6798.4)	341.6 (8152.6)
Mean	1,689,415	1,097,629	411,535	447,891
N	24192	24192	24192	21531

Note: The Table reports estimates of the dynamic effects of an worker death on firms (as in the difference-in-differences model 1). Standard errors clustered at the firm level are reported in parentheses. Stars denote statistical significance: ** p0.01, * p0.05, + p0.1. See Appendix B for information on variables construction.

Table A.4: Effect of Worker Death on Firms: Impact on Balance Sheets and Exceptional Earnings and Hours Worked

Lead /Lag	Capital	Assets	Equity	Export	External Worker	Overtime Hours	Hours Worked	Overtime Earnings	Earnings
-3	-29813.165 (61191.19)	-24543.428 (68206.66)	-15906.311 (33119.12)	1589.257 (1526.80)	10.743 (7.10)	-2.359 (46.08)	473.301 (621.40)	-3860.924 (5166.03)	63492.719 (113303.97)
-2	-14649.335 (39956.14)	-40295.865 (61992.01)	-23848.213 (35817.69)	-1093.368 (1285.09)	5.929 (6.37)	-30.842 (41.86)	733.839 (462.32)	-2279.488 (3893.18)	82046.373 (83735.79)
-1	0.000 (.)	0.000 (.)	0.000 (.)	0.000 (.)	0.000 (.)	0.000 (.)	0.000 (.)	0.000 (.)	0.000 (.)
0	-22708.352 (51103.99)	-36604.507 (63334.75)	-18932.002 (33183.47)	-1135.915 (1302.06)	10.380 (6.48)	20.607 (49.81)	204.562 (447.40)	7570.432 (6085.95)	56594.599 (82924.30)
1	-27483.770 (50669.04)	-62869.276 (67854.73)	-39029.342 (37943.70)	-1150.861 (1471.49)	5.775 (7.15)	19.656 (61.40)	621.451 (550.19)	11609.100* (6548.48)	155075.263 (104950.08)
2	45296.585 (47383.71)	51274.139 (57850.58)	-2341.743 (34478.48)	-1560.966 (1670.58)	4.096 (7.41)	13.688 (62.27)	262.226 (618.34)	10771.264 (6740.07)	97919.089 (118693.37)
3	-22880.909 (54516.57)	-9563.990 (68330.89)	-25445.976 (37469.49)	-2258.523 (1916.05)	10.369 (7.02)	24.875 (65.79)	900.115 (681.06)	15501.147** (7042.31)	200527.944 (135548.59)
N	21531	21531	21531	21531	12074	6942	6942	6942	6942

Note: The Table reports estimates of the dynamic effects of an worker death on firms (as in the difference-in-differences model 1). Standard errors clustered at the firm level are reported in parentheses. Stars denote statistical significance: ** p0.01, * p0.05, + p0.1. See Appendix B for information on variables construction.

Table A.5: Effect of Worker Death on Firms: Heterogeneity by Firm characteristics and Deceased worker characteristics for the outcome variable *number of Employees*

Heterogeneity by...	Entire Sample	Bottom Tercile	Medium Tercile	Top Tercile
Employer Size	-0.325** (0.130)	-0.475* * * (0.106)	-0.346* (0.209)	-0.102 (0.317)
N	15088	5357	4775	4956
Employer Wage	-0.325** (0.130)	-0.505** (0.210)	-0.512** (0.226)	0.0608 (0.241)
N	15088	4955	5042	5091
Employer Productivity	-0.325** (0.130)	-0.661* * * (0.228)	-0.124 (0.235)	-0.156 (0.218)
N	15088	4947	5064	5077
Age of Deceased	-0.325** (0.130)	-0.0985 (0.239)	-0.164 (0.210)	-0.732* * * (0.219)
N	15088	5219	5043	4826
Tenure of Deceased	-0.325** (0.130)	0.0935 (0.248)	-0.409* (0.210)	-0.604* * * (0.215)
N	15088	5127	5021	4940
Earnings of Deceased	-0.325** (0.130)	-0.549* * * (0.202)	-0.381 (0.232)	-0.0439 (0.242)
N	15088	4988	4943	5157

Note: The Table reports estimates of the effects of an worker death on firms number of employees up to year one year after worker death. 'Low' 'Medium' and 'High' represent different specifications where we select the bottom tercile, mid tercile and top tercile respectively. Standard errors clustered at the firm level are reported in parentheses. Stars denote statistical significance: *** p0.01, ** p0.05, * p0.1. See Appendix B for information on variables construction.

Table A.6: Effect of Worker Death on Firms: Heterogeneity by Firm characteristics and Deceased worker characteristics for the outcome variable *Profit*

Heterogeneity by...	Entire Sample	Bottom Tercile	Medium Tercile	Top Tercile
Employer Size	-11071.4 (7540.9)	-23387.0** (10617.2)	-8358.1 (12735.1)	-1606.5 (15019.3)
N	15088	5357	4775	4956
Employer Wage	-11071.4 (7540.9)	-10115.5 (8989.7)	2720.8 (12517.7)	-28115.0* (16211.4)
N	15088	4955	5042	5091
Employer Productivity	-11071.4 (7540.9)	-8955.4 (10430.2)	-5521.1 (11305.7)	-22984.4 (15797.8)
N	15088	4947	5064	5077
Age of Deceased	-11071.4 (7540.9)	-1635.3 (12214.3)	-20452.4 (12603.3)	-13509.4 (14193.7)
N	15088	5219	5043	4826
Tenure of Deceased	-11071.4 (7540.9)	-6557.7 (11574.7)	-27500.6** (13313.5)	-956.9 (13352.3)
N	15088	5127	5021	4940
Earnings of Deceased	-11071.4 (7540.9)	958.9 (12103.7)	-25040.8** (11817.4)	-12914.0 (14513.7)
N	15088	4988	4943	5157

Note: The Table reports estimates of the effects of an worker death on firms profit (measured in euros) up to year $k = 1$. ‘Low’ ‘Medium’ and ‘High’ are specifications where we select the bottom tercile, mid tercile and top tercile respectively. Standard errors clustered at the firm level are reported in parentheses. Stars denote statistical significance: *** p0.01, ** p0.05, * p0.1. See Appendix B for information on variables construction.

Table A.7: Effect of Worker Death on Incumbent Workers: Impact on the Composition of Total Earnings & Total Hours Worked

	Daily Earnings €	# Days Worked	Overtime Hours	Overtime Earnings	Bonus Earnings	Overtime & Bonus Earnings
-3	-0.168 (0.433)	0.0534 (0.669)	3.65 (3.07)	32.17 (37.33)	61.15 (80.61)	93.22 (86.72)
-2	0.0674 (0.353)	0.114 (0.567)	1.43 (3.29)	34.73 (25.70)	34.61 (74.90)	69.29 (78.90)
-1	0 (.)	0 (.)	0 (.)	0 (.)	0 (.)	0 (.)
0	0.986** (0.387)	0.658 (0.794)	-0.08 (2.498)	35.78* (20.80)	32.81 (58.27)	61.60 (61.80)
1	5.182** (2.048)	-1.173 (1.086)	1.492 (3.14)	72.14*** (26.27)	218.2*** (76.91)	291.6*** (82.12)
2	1.298 (1.079)	0.120 (1.113)	1.60 (3.35)	83.41*** (28.21)	211.4*** (81.46)	294.8*** (85.95)
3	2.641* (1.536)	0.472 (1.120)	0.16 (3.24)	95.69*** (33.11)	57.30 (123.7)	152.9 (128.1)
Mean	99.68	359.04	21.7	334.96	1,438	1,773
N	147414	147414	30,902	30,902	30,902	30,902

Note: The Table shows regression coefficients and associated standard errors for the difference between treatment and comparison group in a given year k relative to the death of a worker in the treatment group firms, i.e., the β_k^T from the difference-in-differences model 2. The coefficient in $k = -1$ is normalized to zero. Exponent S indicate that variables are constructed from survey data. Sample includes only incumbent workers that do not switch years (stayers) and observations are weighted inversely by the number of incumbent workers. Standard errors clustered at the worker level are reported in parentheses. Stars denote statistical significance: *** p0.01, ** p0.05, * p0.1. Earnings are daily earnings. Appendix B for information on variables construction. Table A.8 reports estimates for the entire set of incumbent workers (stayers and switchers).

Table A.8: Effect of Worker death on Incumbent Workers: Impact on *All* Incumbent Coworkers (stayers and switchers)

Lead /Lag	Earnings €	<i>Earnings</i> ^S €	Total Hours Worked	Total Hours <i>Worked</i> ^S	Wage €	<i>Wage</i> ^S €	Switch Firm
-3	-57.62 (108.2)	528.5 (337.5)	-6.028 (5.036)	31.46** (13.27)	-0.0274 (0.142)	-0.0630 (0.162)	0.00 (0.00)
-2	-117.2 (82.58)	-2.111 (262.5)	-2.125 (3.838)	20.98* (11.41)	-0.0954 (0.0919)	-0.370*** (0.128)	0.00 (0.00)
-1	0 (.)	0 (.)	0 (.)	0 (.)	0 (.)	0 (.)	0 (.)
0	295.6*** (88.69)	565.4** (264.2)	6.673 (4.075)	21.75** (10.93)	-0.0802 (0.144)	0.00988 (0.138)	0.000 (0.003)
1	544.2*** (125.0)	723.6** (302.3)	6.858 (5.229)	23.05* (12.80)	0.352** (0.172)	0.0340 (0.156)	0.001 (0.004)
2	551.3*** (144.3)	809.4** (339.9)	9.918* (5.691)	23.36* (13.21)	0.163 (0.142)	0.0566 (0.177)	0.002 (0.005)
3	374.7** (158.0)	540.5 (373.3)	3.164 (6.165)	23.90* (14.18)	0.201 (0.154)	-0.112 (0.179)	-0.005 0.006
Mean	32,740	36,715	1,511	1,528	21.51	24.06	0.0
N	234,644	44,891	234,644	44,891	234,644	44,891	234,644

Note: The Table shows regression coefficients and associated standard errors for the difference between treatment and comparison group in a given year k relative to the death of a worker in the treatment group firms, i.e., the β_k^T from the difference-in-differences model 2. The coefficient in $k = -1$ is normalized to zero. Exponent S indicate that variables are constructed from survey data. Sample includes only incumbent workers that *do* and *do not* change employers. Observations are weighted inversely by the number of incumbent workers. Standard errors clustered at the worker level are reported in parentheses. Stars denote statistical significance: *** p0.01, ** p0.05, * p0.1. Appendix B for information on variables construction. Table 3 reports estimates for the set of incumbent stayers workers.

Table A.9: Effect of Worker death on Incumbent Workers: unweighted by Employer Size

Lead /Lag	Total Earnings €	Total <i>Earnings</i> ^S €	Total Hours Worked	Total Hours <i>Worked</i> ^S	Wage €	<i>Wage</i> ^S €
-3	2.901 (102.9)	380.5 (281.0)	-5.229 (3.746)	24.82** (10.26)	-0.0574 (0.126)	-0.192 (0.151)
-2	6.511 (76.89)	-149.3 (228.4)	-0.907 (3.032)	16.49* (8.890)	-0.0334 (0.0856)	-0.342* * * (0.117)
-1	0 (.)	0 (.)	0 (.)	0 (.)	0 (.)	0 (.)
0	141.1* (79.82)	454.9** (184.8)	4.837* (2.882)	22.48* * * (7.178)	0.0466 (0.122)	-0.142 (0.107)
1	241.0** (106.8)	355.4 (217.0)	1.907 (3.737)	7.804 (7.751)	0.114 (0.165)	0.0206 (0.126)
2	152.5 (123.7)	571.3** (251.7)	2.554 (4.089)	11.74 (8.426)	-0.0176 (0.135)	0.0443 (0.146)
3	133.5 (136.8)	162.5 (284.8)	2.538 (4.391)	9.499 (9.100)	-0.0426 (0.135)	-0.101 (0.156)
Mean	35,800	39,308	1,559	1,587	22.9	24.8
N	147414	30902	147414	30902	147414	30902

Note: The Table shows regression coefficients and associated standard errors for the difference between treatment and comparison group in a given year k relative to the death of a worker in the treatment group firms, i.e., the β_k^T from the difference-in-differences model 2. The coefficient in $k = -1$ is normalized to zero. Exponent S indicate that variables are constructed from survey data. Sample includes only incumbent workers that do not switch years (stayers) and observations are *not* weighted inversely by the number of incumbent workers. Standard errors clustered at the worker level are reported in parentheses. Stars denote statistical significance: *** p0.01, ** p0.05, * p0.1. Appendix B for information on variables construction. Table 3 reports estimates for the set of incumbent stayers workers.

Table A.10: Elasticity of Output to Labor Input: **Wage bill** to measure labor input

	OLS		Instrumental Variable		
			Treated Firm Size & Q(Age) & Q(Wage)	IV-LASSO	Treated Firm Size
Baseline: Hours Worked to Measure Labor Input					
Hours Worked	.344*** (.019)	.484*** (.119)	.555*** (.167)	.614*** (.183)	.651* (.386)
F-test IV		3.7	9.3	16.1	3.63
N	3,456	3,456	3,456	3,456	3,456
Wage Bill to Measure Labor Input					
Wage Bill	.382*** (.019)	.470*** (.112)	.633*** (.155)	.734*** (.223)	.829 (.525)
F-test IV		4.01	7.1	11.2	2.2
N	3,456	3,456	3,456	3,456	3,456

Note: The table shows regression coefficients and associated standard errors for model 10 for two different measure of labor input. Columns (2), (3), (4) and (5) use different excluded instruments. We report the baseline specification in which labor input is measured as total hours worked, and the specification using wage bill.

Table A.11: Elasticity of Output to Labor Input: not first-differenced

	OLS		Instrumental Variable			
	OLS	OLS-FE	Treated Firm Size & Q(Age) & Q(Wage)	IV-LASSO	Treated Firm Size	Treated
	.199*** (.016)	.083*** (.014)	.268*** (.028)	.285*** (.029)	.280*** (.036)	.323 (.281)
F-test IV			88.5	121.1	450.7	4.5
N	24,192	24,192	24,192	24,192	24,192	24,192

Note: The table shows regression coefficients and associated standard errors for model 10, where output and labor input are not converted to growth rate (log first difference). Columns (2), (3), (4) and (5) use different excluded instruments.

Table A.12: Hours Worked and Earnings Elasticity to Firm Size: All **Coworkers** and Firm-Level specifications

	OLS	Instrumental Variable			
		$\frac{\text{Treated}}{\text{Firm Size}}$ & $\frac{\text{Treated}}{\text{Firm Size}}$	IV-LASSO	$\frac{\text{Treated}}{\text{Firm Size}}$	Treated
Q(Age) & Q(Wage)					
a) Baseline: Stayer Workers					
b_1	-.029**	-.136*	-.180**	-.434**	-.267**
s.e	(.012)	(.075)	(.085)	(.169)	(.130)
Labor supply elasticity	.07	.40	.57	1.98	.94
$c_1 = \text{Earnings}$	-.007	-.263***	-.323***	-.774***	-.570***
s.e	(.012)	(.089)	(.099)	(.229)	(.162)
$c_1 = \text{Hourly wage}$.024**	-.117	-.132	-.315*	-.279*
s.e	(.010)	.78	(.086)	(.163)	(.126)
F-test IV		13.6	35.8	24.0	45.7
N	20,178	20,178	20,178	20,178	20,178
b) All Coworkers (Stayers and Switchers)					
b_1	-.010	-.221***	-.222***	-.381***	-.203*
s.e	(.009)	(.078)	(.083)	(.136)	(.110)
Labor supply elasticity	.02	.73	.74	1.6	.65
$c_1 = \text{Earnings}$	-.002	-.325***	-.343***	-.587***	-.473***
s.e	(.011)	(.094)	(.099)	(.169)	(.142)
$c_1 = \text{Hourly wage}$.009	-.100	-.116	-.196	-.259**
s.e	(.009)	(.072)	(.076)	(.121)	(.102)
F-test IV		21.4	46.5	49.0	88.5
N	31,952	31,952	31,952	31,952	31,952
c) Average Hours Worked and Average Earnings (Firm-Level)					
b_1	.007	-.079	-.023	-.263*	-.202
s.e	(.012)	(.061)	(.064)	(.142)	(.131)
Labor supply elasticity	-.01	.22	.06	.92	.65
$c_1 = \text{Earnings}$.035***	-.118	0.057	-.568***	-.489**
s.e	(.012)	(.080)	(.087)	(.216)	(.190)
$c_1 = \text{Hourly wage}$.028***	-.040	0.080	-.305*	-.286*
s.e	(.010)	(.080)	(.084)	(.167)	(.152)
F-test IV		4.6	30.5	14.68	15.83
N	3,334	3,334	3,334	3,334	3,334

Note: The table shows regression coefficients and associated standard errors for models 11 and 12 a) for ‘stayers’ coworkers, b) all coworkers (stayers and movers) and c) Firm-Level specifications . Columns (2), (3), (4) and (5) use different excluded instruments.

Table A.13: Hours Worked and Earnings Elasticity to Firm Size: not first-differenced

	OLS		Instrumental Variable			
	OLS	OLS-FE	Treated Firm Size & Q(Age) & Q(Wage)	IV-LASSO	Treated Firm Size	Treated
$b_1 = \epsilon_{h/N}$	-.034***	-.018***	-.040**	-.040**	-.043**	.101
s.e	(.005)	(.006)	(.016)	(.016)	(.019)	(.498)
LS elasticity	.04	.02	.05	.05	.06	-.12
$c_1 = \epsilon_{w/N}$	-.033***	.008	-.031	-.024	-.029	.650
s.e	(.008)	(.006)	(.026)	(.026)	(.032)	(.983)
F-test IV			266.4	620.4	990.1	1.3
N	147,414	147,414	147,414	147,414	147,414	147,414

Note: The table shows regression coefficients and associated standard errors for models 11 and 12, where hours worked and earnings are not converted to growth rate (log first difference). Columns (2), (3), (4) and (5) use different excluded instruments.

D Supplementary Tables on Sample Description

Table A.14: Causes of death

Causes of death	Count	ICD codes
Accidents	227	8,V,W,X0-X5
Suicides	98	95,X6-X7, X80-X84
Heart Diseases	158	I,39-45
Other Unexpected	648	R,S,T,X8,X9,Y,90-99,Y
Other	653	
Total	1728	

Table A.15: Firm-Level Summary Statistics on the last 3 years before the event

	Comparison Firms		Treated Firms		p-value
	Mean	sd	Mean	sd	
# Employee	12.92	6.73	12.93	6.75	0.68
# Hours Work	18,776.47	10,348.47	18,741.37	10,528.49	0.74
# Hires	7.44	8.71	7.32	7.83	0.24
# Separations	6.75	8.19	6.66	7.40	0.36
Value Added	658,279.93	456,986.38	658,020.20	470,739.86	0.98
Labor Cost	440,431.68	280,617.53	438,362.46	277,746.64	0.57
Profit	217,502.16	286,885.20	218,822.99	314,738.29	0.87
Capital	546,550.41	1,675,137.40	600,721.88	2,413,707.98	0.15

Note: The table reports summary statistics for the treated and comparison group in the last 3 years before the event. The table illustrates that treated and comparison firms are similar. 'sd' refers to standard deviation and 'p-value' reports the p-value of the t-test of the statistical significance of the treatment status in mean outcomes. Section 2.3 defines the treated and comparison group and the sample restriction.

Table A.16: Coworker-Level Summary Statistics on the last 3 years before the event

	Comparison Firms		Treated Firms		p-value
	Mean	sd	Mean	sd	
Earnings	34,703.00	14503	34,174.82	14468	0.01**
# Hours worked	1,532.20	332	1,526.39	343	0.09*
Wage	22.69	9.9	22.43	12	0.05**
Age of workers	43.39	12	43.03	13	0.00***
Education	12.63	2.6	12.56	2.7	0.01**
Men	0.73	.44	0.72	.45	0.13
Survey data					
Earnings	37,978.43	13900	37,505.02	14174	0.05**
Hours worked	1,538.02	365	1,513.07	367	0.02**
Wage	24.87	8	24.88	8	0.95
Overtime hours	24.91	128	19.99	86	0.10*
Overtime Earnings	372.09	1410	351.46	1413	0.61
Bonus Earnings	1,388.51	3329	1,329.24	3469	0.57

Note: Summary statistics are measured one year before the event for incumbent workers that do not switch firms (Incumbent-Stayer). Danish Kroner are deflated and converted to EUR. 'sd' refers to standard deviation. 'p-value' is the p-value of t-test of the significance of the treatment status on the mean outcome. See Appendix B for information on variables construction. Table A.2 reports statistics computed on the last year before the event for Incumbent-Stayers. Table A.17 reports summary statistics the entire set of Incumbent workers (stayers and switchers).

Table A.17: Coworker-Level Summary Statistics: All Coworkers

	Comparison Firms		Treated Firms		p-value
	Mean	sd	Mean	sd	
Earnings	32,342.43	14689	32,196.90	14454	0.60
# Hours worked	1,499.58	388	1,504.45	387	0.40
Wage	21.43	9.5	21.36	9.9	0.69
Age	41.07	13	40.77	13	0.10
Education	12.84	2.6	12.77	2.6	0.11
Men	0.73	.44	0.73	.44	0.85
Hours worked (Survey)	1,466.19	439	1,438.29	445	0.11
Overtime hours	25.59	101	22.85	84	0.57
Earnings (Survey)	34,862.40	14747	34,681.29	14920	0.74
Overtime Earnings	398.95	1385	371.15	1371	0.68
Bonus Earnings	1,169.23	3074	1,088.51	2983	0.62
Wage (Survey)	23.86	7.9	24.15	8.2	0.43

Note: Summary statistics are measured one year before the event for the entire set of incumbent workers (stayers and switchers). Danish Kroner are deflated and converted to EUR. ‘sd’ refers to standard deviation. ‘p-value’ is the p-value of t-test of the significance of the treatment status on the mean outcome. Table A.2 and A.16 refers to the set of incumbent workers that do not switch firms (Incumbent-Stayer).

E Model Derivation and Extensions

Model of Labor supply

We use a basic one period labor supply model. The only decision for workers is the intensive-margin. There are no frictions or adjustment costs. Consumption in a period is given by the static budget constraint $C_t = w_t h_t + Y_t$. The preferences of workers over hours worked h and consumption c of the final good are represented by

$$u(c, h) = c - \phi(h), \quad (\text{A.1})$$

we use the utility function $\phi(h) = \frac{h^{1+\eta}}{1+\eta}$, $\eta \geq 0$, represent the disutility of labor. As u is quasilinear, the nonlabor income (Y_t) does not matter. This implies that the compensated (Hicksian) labor supply is equal to the uncompensated (Marshallian) labor supply. The optimal hours worked for the worker is $h = w^{-\eta}$. The elasticity is $\epsilon \equiv \frac{dh}{dw} \frac{w}{h} = \frac{1}{\eta}$. Chetty et al. (2011) show how adjustment costs affect estimates of the intensive margin elasticity of labor supply.

Nash Bargaining for Hours Worked and Wage

We use the generalized Nash solution to find bargained wage (w) and hours worked (h). First it can be shown that the maximisation of the generalized Nash solution with respect to w yield the following sharing rule for the wage:

$$(1 - \beta) [W(N) - U] = \beta \Pi'(N), \quad (\text{A.2})$$

Hours Worked. To derive the Nash solution with respect to h , we proceed as follow. By definition, the surplus of the marginal job is:

$$S(N) = W(N) - U + \Pi'(N),$$

which can be written as:

$$\begin{aligned} rS(N) &= R'(Nh(N)) [h(N) + Nh'(N)] - Nw'(N) - \phi(h(N)) \\ &\quad - qS(N) + [mV(N) - qN] [W'(N) + \Pi''(N)] + \dot{S}(N). \end{aligned}$$

The optimal bargaining solution implies that w and h are chosen to share the current surplus in every period. For any value of w , there is a value of h which maximizes the surplus. It is optimal to choose this value of h and to bargain on the wage to share this surplus, which has been maximized with respect to h .

The expression $S(N)$ given above defines the marginal surplus as function of N when $h = h(N)$. To determine the optimal choice of h we have to write the surplus of the marginal worker as a function of their hours of work, which can be written as:

$$S(N) = \Pi(N) - \Pi(N - 1) + W(N) - U,$$

or, using the previous equations:

$$\begin{aligned} r[\Pi(N) - \Pi(N - 1) + W(N) - U] &= \max_h R((N - 1)h(N) + h) - Nw(N) - qN\Pi'(N) \\ &\quad - C(V(N)) + mV(N)\Pi'(N) + \dot{\Pi}(N) \\ &\quad - R((N - 1)h(N - 1)) - (N - 1)w(N - 1) - q(N - 1)\Pi'(N - 1) \\ &\quad - C(V(N - 1)) + mV(N - 1)\Pi'(N - 1) + \dot{\Pi}(N - 1) \\ &\quad + w(N) - \phi(h) + q(U - W(N)) + (mV - qN)W'(N) + \dot{W}(N) - rU. \end{aligned}$$

In the first row of this equation, the term $(N - 1)h(N) + h$ represents the total number of hours worked by the $N - 1$ inframarginal workers when there are N workers in the firm, who each work $h(N)$ hours. Therefore, the revenue produced by the hours of work of the marginal worker is equal to $R((N - 1)h(N) + h) - R((N - 1)h(N - 1))$.

Now, the first order condition with respect to h implies that:

$$R'((N-1)h(N) + h(N)) = \phi'(h).$$

Since $R'((N-1)h(N) + h(N)) = R'(Nh(N))$, we have:

$$R'(Nh(N)) = \phi'(h).$$

Determination of Wage. Use the worker HJB equation, which is:

$$(r+q)[W(N) - U] = w(N) - rU - \phi(h(N)) + [mV(N) - qN]W'(N) + \dot{W}(N), \quad (\text{A.3})$$

and the marginal job HJB equation,

$$(r+q)\Pi'(N) = R'(Nh(N))(h(N) + Nh'(N)) - w(N) - Nw'(N) + [mV(N) - qN]\Pi''(N) + \dot{\Pi}'(N). \quad (\text{A.4})$$

Using the sharing rule and the two previous equations, we have

$$(1-\beta)[w(N) - rU - \phi(h(N)) + [mV(N) - qN]W'(N) + \dot{W}(N)] = \beta[R'(Nh(N))(h(N) + Nh'(N)) - w(N) - Nw'(N) + [mV(N) - qN]\Pi''(N) + \dot{\Pi}'(N)]$$

Or,

$$w(N) = (1-\beta)[rU + \phi(h(N))] + \beta[R'(Nh(N))(h(N) + Nh'(N)) - Nw'(N)] + \beta[[mV(N) - qN]\Pi''(N) + \dot{\Pi}'(N)] - (1-\beta)[[mV(N) - qN]W'(N) + \dot{W}(N)]$$

or

$$w(N) = (1-\beta)[rU + \phi(h(N))] + \beta[R'(Nh(N))(h(N) + Nh'(N)) - Nw'(N)] + [mV(N) - qN][\beta\Pi''(N) - (1-\beta)W'(N)] + \beta\dot{\Pi}'(N) - (1-\beta)\dot{W}(N)$$

The derivative of the sharing rule with respect to N yields

$$W'(N)(1-\beta) = \beta\Pi''(N).$$

The derivative of the sharing rule with respect to time yields

$$(1-\beta)[\dot{W}(N) - \dot{U}] = \beta\dot{\Pi}'(N). \quad (\text{A.5})$$

Using these 2 equations and the previous expression of the wage, we get

$$w(N) = \beta [R'(Nh(N)) [h(N) + Nh'(N)] - Nw'(N)] + (1 - \beta) \phi(h(N)) + B,$$

where $B = (1 - \beta) (rU - \dot{U})$.

We can express the wage as function of N using the bargained hours of work:

$$\alpha AN^{\alpha-1} = h^{1+\eta-\alpha} \Leftrightarrow h = (\alpha A)^{\frac{1}{1+\eta-\alpha}} N^{\frac{\alpha-1}{1+\eta-\alpha}} \Rightarrow h'(N) = \frac{\alpha-1}{1+\eta-\alpha} \frac{h}{N}$$

and the definition of

$$hR'(Nh) = A\alpha h^\alpha N^{\alpha-1} = A\alpha (\alpha A)^{\frac{\alpha}{1+\eta-\alpha}} N^{\frac{(\alpha-1)\alpha}{1+\eta-\alpha}} N^{\alpha-1} = (\alpha A)^{\frac{(1+\eta)}{1+\eta-\alpha}} N^\gamma$$

where $\gamma = \frac{(\alpha-1)(1+\eta)}{1+\eta-\alpha}$

and

$$\begin{aligned} R'(Nh(N))Nh'(N) &= \frac{\alpha-1}{1+\eta-\alpha} A\alpha h^{\alpha-1} N^{\alpha-1} N \frac{h}{N} = \frac{\alpha-1}{1+\eta-\alpha} A\alpha h^\alpha N^{\alpha-1} \\ &= \frac{\alpha-1}{1+\eta-\alpha} A\alpha (\alpha A)^{\frac{\alpha}{1+\eta-\alpha}} N^{\frac{(\alpha-1)\alpha}{1+\eta-\alpha}} N^{\alpha-1} \\ &= \frac{\alpha-1}{1+\eta-\alpha} (\alpha A)^{\frac{(1+\eta)}{1+\eta-\alpha}} N^\gamma \end{aligned}$$

so that

$$[h(N) + Nh'(N)] R'(Nh(N)) = \frac{\eta}{1+\eta-\alpha} (\alpha A)^{\frac{(1+\eta)}{1+\eta-\alpha}} N^\gamma$$

We get

$$\begin{aligned} w(N) &= B + (1 - \beta) \frac{(\alpha A)^{\frac{(1+\eta)}{1+\eta-\alpha}} N^\gamma}{1+\eta} + \beta \left[\frac{\eta}{1+\eta-\alpha} (\alpha A)^{\frac{(1+\eta)}{1+\eta-\alpha}} N^\gamma - Nw'(N) \right] \\ w(N) &= B + (\alpha A)^{\frac{(1+\eta)}{1+\eta-\alpha}} N^\gamma \left[\frac{1-\beta}{1+\eta} + \frac{\beta\eta}{1+\eta-\alpha} \right] - \beta Nw'(N) \\ w(N) &= B + \beta (\alpha A)^{\frac{(1+\eta)}{1+\eta-\alpha}} N^\gamma \left[\frac{1-\beta}{\beta(1+\eta)} + \frac{\eta}{1+\eta-\alpha} \right] - \beta Nw'(N) \end{aligned}$$

or

$$\begin{aligned} w(N) &= B + \beta [\Phi(N) - Nw'(N)] \\ \text{where } \Phi(N) &= \gamma_0 N^\gamma, \gamma_0 = (\alpha A)^{\frac{(1+\eta)}{1+\eta-\alpha}} \left[\frac{1-\beta}{\beta(1+\eta)} + \frac{\eta}{1+\eta-\alpha} \right] \end{aligned}$$

The solution has already been studied in Cahuc et al. (2008).¹² We get:

$$\begin{aligned} w(N) &= B + N^{-1/\beta} \int_0^N z^{\frac{1-\beta}{\beta}} \Phi(z) dz \\ w(N) &= B + \gamma_0 N^{-1/\beta} \int_0^N z^{\gamma + \frac{1-\beta}{\beta}} dz \\ w(N) &= B + \gamma_0 N^\gamma \int_0^1 z^{\gamma + \frac{1-\beta}{\beta}} dz. \end{aligned}$$

Using equation (6), we can also rewrite the wage as:

$$w(N) = (1 - \beta) (rU - \dot{U}) + \phi h^{1+\eta},$$

$$\text{where } \phi = \frac{\beta}{1+\beta\gamma} \left[\frac{1-\beta}{\beta(1+\eta)} + \frac{\eta}{1+\eta-\alpha} \right]$$

We can also compute the hourly wage, $\omega(N) = w(N)/h(N)$,

Hiring cost as function of labor inputs

In steady state we have

$$\frac{C'(V)}{m} = \frac{\alpha A [Nh(A, N)]^{\alpha-1} [h(A, N) + Nh_N(A, N)] - w(A, N) - Nw'(A, N)}{r + q}$$

This equation can also be written

$$\frac{C'(V)}{ym} = \frac{\alpha \frac{\partial(Nh)}{\partial N} - \frac{1}{y} \frac{\partial(Nw)}{\partial N}}{r + q}$$

where $y = Y/Nh$ is the value added per hour of work.

We have, for each firm:

$$Nh = (\alpha A)^{\frac{1}{1+\eta-\alpha}} N^{\frac{\eta}{1+\eta-\alpha}} \implies \frac{\partial(Nh)}{\partial N} = \frac{\eta}{1+\eta-\alpha} h \quad (\text{A.6})$$

$$Nw(A, N) = (1 - \beta) (rU - \dot{U}) N + \frac{\beta\gamma_0}{1 + \beta\gamma} N^{\gamma+1} \implies \frac{\partial(Nw)}{\partial N} = w(A, N) + \frac{\beta\gamma_0}{1 + \beta\gamma} \gamma N \quad (\text{A.7})$$

Therefore, we get

$$\frac{C'(V)}{ym} = \frac{1}{r + q} \left(\frac{\alpha\eta}{1 + \eta - \alpha} h - \frac{1}{y} \left(w + \frac{\beta\gamma_0}{1 + \beta\gamma} \gamma N \right) \right)$$

with $\gamma = \frac{(\alpha-1)(1+\eta)}{1+\eta-\alpha}$; $\gamma_0 = (\alpha A)^{\frac{(1+\eta)}{1+\eta-\alpha}} \left[\frac{1-\beta}{\beta(1+\eta)} + \frac{\eta}{1+\eta-\alpha} \right]$; $y = Y/Nh$

¹²See Petrosky-Nadeau and Wasmer (2017) for a sketch of a solution.

Decomposition of the Hiring Cost

Decomposition of the Marginal Value of a job

We start from

$$\Pi_N(A, N) = \frac{\alpha A [Nh(A, N)]^{\alpha-1} [h(A, N) + Nh_N(A, N)] - w(A, N) - Nw'(A, N)}{r + q}$$

which accounts for the adjustment of hours of work and of wages.

1. Assuming that there is no adjustment of hours of work and no adjustment of wages, the marginal value of jobs is

$$\begin{aligned} \Pi_N(A, N) &= \frac{\alpha A [Nh(A, N)]^{\alpha-1} h(A, N) - w(A, N)}{r + q} \\ &= \frac{\alpha y h(A, N) - w(A, N)}{r + q} \end{aligned}$$

2. Assuming that there is no adjustment of hours of work but adjustment of wages, the marginal value of jobs is

$$\begin{aligned} \Pi_N(A, N) &= \frac{\alpha A [Nh(A, N)]^{\alpha-1} h(A, N) - w(A, N) - Nw'(A, N)}{r + q} \\ &= \frac{\alpha y h(A, N) - \left(w(A, N) + \frac{\beta \gamma_0}{1 + \beta \gamma} \gamma N \right)}{r + q} \end{aligned}$$

3. Assuming that there is no adjustment of wages of work but adjustment of hours, the marginal value of jobs is

$$\begin{aligned} \Pi_N(A, N) &= \frac{\alpha A [Nh(A, N)]^{\alpha-1} [h(A, N) + Nh_N(A, N)] - w(A, N)}{r + q} \\ &= \frac{\alpha \frac{\eta}{1 + \eta - \alpha} y h(A, N) - w(A, N)}{r + q} \end{aligned}$$

Model with Stochastic Productivity Parameter

Here we extend the basic model to allow for stochastic productivity. Otherwise the model is unchanged.

Production technology. A is a productivity parameter which evolves according to the Brownian motion with drift $\mu > 0$ and variance $\sigma^2 > 0$:

$$dA = \mu dt + \sigma dB$$

where B is a standard Weiner process.

Bargaining over wages and hours. In this context, the generic problem of the firm is

$$\max_{\tau, (V_t)_{t \leq \tau}} \mathbb{E}_0 \int_0^\tau \pi(A_t, N_t, V_t) e^{-rt} dt$$

where

$$\begin{aligned} \pi(A_t, N_t, V_t) &= A_t [N_t h(A_t, N_t)]^\alpha - N_t w(A_t, N_t) - C(V_t) \\ \dot{N}_t &= mV_t - qN_t \end{aligned}$$

We now define the value functions of firms and of workers to determine employment, hours worked and wages. We denote by $\Pi(A, N)$ the **value of the firm**, which satisfies the Hamilton-Jacobi-Bellman equation:

$$r\Pi(A, N) = A [Nh(A, N)]^\alpha - Nw(A, N) - qN\Pi_N(A, N) + \max_V [-C(V) + mV\Pi_N(A, N)] + \mu\Pi_A(A, N) + \frac{\sigma^2}{2}\Pi_{AA}(A, N) \quad (\text{A.8})$$

where Π_x denotes the derivative of Π with respect to its argument x , and the Hamilton-Jacobi-Bellman variational inequality

$$\begin{aligned} \min\{ & r\Pi(A, N) - A [Nh(A, N)]^\alpha + Nw(A, N) + qN\Pi_N(A, N) \\ & - \max_V [-C(V) + mV\Pi_N(A, N)] - \mu\Pi_A(A, N) - \frac{\sigma^2}{2}\Pi_{AA}(A, N), \Pi(A, N)\} = 0 \end{aligned} \quad (\text{A.9})$$

From equation (A.8), we deduce that the **optimal number of vacancies**,

$$V(A, N) = \{V | C'(V) = m\Pi_N(A, N)\}, \quad (\text{A.10})$$

satisfies the first order conditions, implying that the marginal cost of job vacancies $C'(V)$ equals the marginal value of a filled job $\Pi_N(A, N)$ times the per-unit-of-time probability to fill vacant jobs.

The **marginal value of a filled job** is computed from the derivative of $\Pi(A, N)$ with respect to N defined by equation (A.8):

$$\begin{aligned} r\Pi_N(A, N) &= \alpha A [Nh(A, N)]^{\alpha-1} [h(A, N) + Nh_N(A, N)] - w(A, N) - Nw_N(A, N) \\ &+ [mV(A, N) - qN] \Pi_{NN}(A, N) - q\Pi_N(A, N) + \mu\Pi_{AN}(A, N) + \frac{\sigma^2}{2}\Pi_{AAN}(A, N) \end{aligned} \quad (\text{A.11})$$

The determination of hours and wage are unchanged in the case with stochastic productivity.

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